### ALGORITHM STUDIES FOR RADAR AND LIDAR SYSTEMS

Hajime Okamoto<sup>\*1)</sup> Suginori Iwasaki<sup>2)</sup> and Hiroaki Horie<sup>1)</sup>
1) Communications Research Laboratory
2) Kobe University

## **1. INTRODUCTION**

Clouds are known to affect the energy and water cycles in the climate system. However there are large uncertainties in the vertical profiles of clouds and this might lead to the large errors in the estimation of longwave cloud radiative forcing at the earth surface. Basically passive instrument can not provide such information with high accuracy and only cloud profiling radar and lidar on a satellite are expected to improve this situation. 95GHz cloud radar enables us to achieve the information of multilayered clouds because the radar-wavelength is normally longer than the sizes of the cloud particles and thus there are small attenuation in the radar signals except for thick water clouds. Lidar instrument is also effective to retrieve the cloud boundaries for the cloud with moderate thickness.

However the single use of radar or lidar only gives us a limited information; for example, it is difficult to retrieve the ice/liquid water content (IWC or LWC) only from radar signal because of the wide variety of size distributions of clouds. Main objective of this paper is to develop the algorithm by use of 95GHz cloud radar and lidar systems for cloud studies. For the analysis

of radar signals, we apply discrete dipole approximations (DDA) in order to take into account the effect of non-sphericity of ice crystals on their backscattering signatures and make a look up table for various size distributions and shapes for the first time with a confidence of accuracy in section 2. In section 3, we discuss about lidar signals. At first, ray tracing technique is tested to estimate backscattering signature for ice crystals at lidar wavelength. It turns out the simple application of ray tracing method to estimate backscattering cross section is problematic for ice crystals with flat surface. We develop the reliable methods to overcome the problem. This also leads to the modification to usual form of lidar equations. In section 4, we describe the retrieval algorithm for the combinational use of radar and lidar signals. And we finally provide the diagram describing the detection-limit for radar and lidar systems in terms of mode radius and IWC. Since February 2, 2000, we have performed simultaneous observations taken by these systems. An example is shown in a last part of this article.

# 2. DDA CALCULATIONS FOR RADAR SIGNALS

For 95GHz cloud radar, the discrete dipole approximation (DDA) is an ideal method to calculate the scattering by non-spherical particles since the size of the particle is small or compa-

<sup>\*</sup> Corresponding author address: Hajime Okamoto, Communications Research Laboratoruy, Kashima Space Research Center, 893-1, Hirai, Kashima, Ibaraki 314-0012, Japan, e-mail: hokamoto@crl.go.jp

rable to the wavelength of interest. DDA is simply described as "it consists of approximating the actual target by an array of polarizable point (the "dipoles")" (Draine 1988). Since this method is an approximation theory, the solutions always contain some errors even if we know the exact shape, size, orientation and refractive indices. Accuracy of this method is tested for the backscattering cross sections for hexagonal ice crystals and for the fixed orientation, i.e., the number of geometry between a particle and incident electromagnetic field is taken to be one, it is found DDA does not work well and its errors for co-. and cross-polarization signals sometimes exceed 100% in the resonance region as long as we use practically possible number of dipoles such as the order of 100000 to mimic the target shape in fig. 1a. The theoretical procedure are established by Okamoto et al., (1995). In realistic atmospheric condition, some kind of averaging could be assumed for the particle orientations. When we assume a particle is oriented randomly in a horizontal plane, where longest axis is parallel to the horizontal plane (hereafter 2D case), DDA turns out to work very fine. The errors in co-polarization signal are less than 5% as shown in fig.1b and those in cross-polarization are also the same order except for plate.





Fig.1a. Errors in backscattered radiance for copol. signal from hexagonal ice crystals with aspect ratio=3 and fixed orientation is assumed. Number corresponds to dipoles used in DDA. 1b. Those for 2D random orientation.

Fig.2a Size integrated radar reflectivity factors by hexagonal ice crystals with several mode radius and aspect ratios for a given IWC=1g/m<sup>3</sup>. Log-normal size distribution is assumed. 2b. Same as a but Linear depolarization ratio. Since cross-polarization signal for a plate with 2D case is very weak, it is not possible to achieve the convergent solutions. In fig.2a and b, on the basis of the error analysis for 2D case, size integrated radar reflectivity factors (dBZ) and linear depolarization ratios (LDR) are estimated for ice crystals with IWC=1g/m<sup>3</sup> by assuming log-normal size distributions with various volume equivalent mode radius re and aspect ratios, e.g., aspect ratio 3 denotes hexagonal column and a=1/3 denotes hexagonal plate. When the minimum separation between co-pol. and cross-one is -30dB, it is possible to discriminate the particle shape from LDR. Note that the values for plate are expressed as upper limit in fig.2b and since these upper limits are much smaller than the detection limit and not a practical problem.

# 3. THEORETICAL ESTIMATES FOR LI-DAR SIGNALS

For lidar wavelength (0.532µm), ray tracing technique is widely used to treat non-sphericity since size of the particle is much larger than the wavelength and thus geometrical approximation can be applied (e.g. Macke 1993). Here we show simple application of this technique is problematic for the estimation of backscattering cross section of the particle with flat surfaces. That is, backscattering cross section is not constant when the angular bin at the backward direction is changed and thus backscattering cross section can not be accurately estimated (fig. 3). This can be naturally understood by the following explanation. For plate with 2D case, backscattered ray is always parallel to the incident ray and thus C<sub>bk</sub> becomes infinity. Instead, we may estimate the reflectance Re of the plate by introducing an approximation that Re of the hexagonal plate is

approximated as that for a slab with a finite thickness depending on the size of the particle and aspect ratio. For this slab, the exact solution is obtained. We call this Finite Slab Approximation (FSA).



Fig.3 Phase function at the backward direction as a function of angular bin used calculated by ray tracing method. The particle is taken to be plate oriented randomly in horizontal plane.



Fig.4 Reflectance of a hexagonal plate with aspect ratio=0.1 oriented randomly in horizontal plane by Finite Slab Approximation (FSA). For comparison, reflectance of a plane with ice for normal incident is illustrated.

The relationship between  $C_{bk}$  and Re is given in eq. (1), where  $\delta$  is the Kronecker's delta function where G(r) is geometric cross section for its radius r. Once  $C_{bk}$  is given for the case of plate with 2D, then by substituting the term expressed in eq. (1) into the general form of lidar equation (2), we can derive the proper lidar equation in eq. (3), where n(r) denotes size distribution and T(z) is transmittance at range z. Here  $\sigma_{bk,e}$  is written in eq. (4) and the effective backscattering coefficient is defined as  $\sigma_{bk,e}$ . In this case, we found there is no dependence of the range z in the received lidar signal.

For hexagonal column with 2D case, it is also numerically shown that  $C_{bk}$  is not converged in the limit of  $\delta\theta$  becomes 0. However this time, the term in eq.(5) is estimated by ray-tracing method and turns to be constant in Fig.5. Substitution of eq. (5) into eq. (2) leads to another dependence of range, proportional to z<sup>-1</sup>, in eq. (6), where s is radius of telescope. The effective backscattering coefficient  $\sigma_{bk,e}$  is also defined in eq. (7). For the hexagonal particles oriented randomly in three dimensional plane (3Dcase), the z-dependence turns to be the same as for the hexagonal column with 2D case by ray tracing method. Note that usual dependence  $z^{-2}$  is usually correct for spherical particles since C<sub>bk</sub> is constant and this z<sup>-2</sup> dependence is naturally derived also from the general form of lidar equation (1). On the basis of previous estimations, we can now calculate the size integrated backscattering coefficients for 2D case in a similar matter as in the radar for a given IWC=1g/  $m^3$  in fig.6.

$$\frac{dC_{sca}(\theta = 180^{\circ})}{d\Omega} = \frac{G(r)R(r,a)\delta_{\theta,180}}{d\Omega}$$
(1)  

$$P_{r}(Z) = P_{r}\int_{v} dv \iiint_{\theta,\varphi,r} \frac{dC_{sca}(r,z)}{d\Omega} \frac{dn(r,z)}{dr} dr d\Omega T(z)^{2}$$
(2)  

$$P_{r} = P_{r}\int dv \cdot \sigma_{kh} T(z)^{2}$$
(3)

$$\sigma_{bk,e} = \int_{r} R_{e}(r)G(r)\frac{dn(r)}{dr}dr \qquad (4)$$

$$\frac{1}{Csca} \frac{dCsca(\theta = 180^{\circ})}{d\theta} = const.$$
 (5)

$$p_r = 2p_t \int dv \cdot \frac{s}{z} \sigma_{bk,e} T(z)^2$$
(6)

$$\sigma_{bk,e} = \int const.G(r) \frac{dn(r)}{dr} dr$$
(7)



Angular bin [deg.]

Fig.5 Differential backscattering cross section estimated within delta  $\theta$  for hexagonal column with 2D case estimated by ray tracing method.





Fig. 6 Size integrated effective lidar backscattering coefficient by hexagonal ice for IWC=1g/m<sup>3</sup>

#### 4. RADAR AND LIDAR ALGORITHM

For a given IWC, the dependence of radius in the radar signal is r<sup>3</sup> and that in the lidar backscattering coefficient is r<sup>-1</sup>. Because of the difference in r-dependence, we might expect to retrieve the size and IWC information by using two active sensors. In fig. 7 the ratio of radar reflectivity factor Ze to lidar backscattering coefficient is derived for the same mode radius and aspect ratio. Surprisingly there are no significant changes in the ratios for different shapes. Once we determine the mode radius, we may derive the IWC from observed radar reflectivity shown in fig. 8.

Applicability of above algorithm is actually restricted to the single layered cloud with thin thickness. However, the extinction of the algorithm is rather straight forward for multi-layered and/or geometrically thick clouds. In addition to the backscattering coefficients, extinction coefficients is also uniquely determined for one set of mode radius and IWC. Therefore by using these look-up tables, we might retrieve mode radius and IWC form these two active sensors.



Mode radius [µm]

Fig. 7 ratio between radar reflectivity factor Ze and lidar backscattering coefficient for the ice crystal with 2D case.



Fig. 8 The relationship between Radar reflectivity factor dBZ and IWC for given mode radius.

Finally the sensitivity of the radar and lidar systems are estimated in terms of mode radius and IWC in Fig. 9. To estimate the minimum requirements of IWC as a function of mode radius, we assumed the radar detection limit is -40dBZ and the detection limit for lidar backscattering coefficient is 10<sup>-4</sup> [km<sup>-1</sup>]. Obviously lidar is more sensitive to small particles compared to radar but radar can have a better sensitivity for large particles. Again the diagram is only directly applicable for the case of single thin layered clouds. For multi-layered and/or optically thick clouds, the sensitivity of lidar greatly reduced, while those for radar is weakly affected.

# 5. SOME EXAMPLES FOR OBSERVA-TIONS

Since 2nd of February, we performed simultanious observations of clouds by co-located radar, radar and microwave radiometer in Kashima, Japan. Both active instruments successfully detects multi-layerd clouds (in fig.10a for radar and fig.10b for lidar, respectively).



Fig. 9 Diagram describing the detection-limits for radar and lidar systems in terms of mode radius and IWC. a1 denotes aspect ratio to be 1.

#### 6. SUMMARY

Principal findings are as follows.

(1) We develop the algorithm for 95GHz cloud radar and lidar systems for the retrievals of microphysics, mainly mode radius, IWC and shape.
(2) For the 95GHz radar, DDA is an ideal tool. The errors are about 5% for both co- and crosspol. signals except for cross-pol.for plate when orientational averaging can be expected.



Fig.10a Radar reflectivity factor by a clouds observaed in Kashima, on Feb. 9 for the period of 9:55-10:33.

(3) For lidar scattering, simple application of ray tracing method turns to be problematic for the particle with flat surfaces. We develop finiteslab approximation for platelike particle with 2D case to overcome this problem. For column with 2D and 3D, ray tracing technique can be applied but to estimate effective backscattering coefficient. Lidar equation also has to be modified and range dependence turns to be now  $z^{-0}$  for plate with 2D and  $z^{-1}$  for column with 2D contrary to the usual dependence of  $z^{-2}$ .

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(4) Algorithm to retrieve mode radius and IWC by using both active sensors is developed.

(5) Diagram for detection-limits for both system is made.

#### References

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10b. Lidar backscattering coefficients by the same clouds.