26A4 SCALE INVARIANT PROPERTIES OF WATER VAPOR MEASURED BY A RAMAN LIDAR

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INTRODUCTION

Data analysis usually involves calculating statistics, such as the mean and the variance of the data. These statistics can be compared to the statistics of other datasets. If these characteristics agree, then the datasets could have come from the same probability distribution function (pdf). The mean is also called the first moment, since it is computed from the data carried to the first power. The variance is the second centered moment computed from deviations of the data from its mean and carried to the second power. Skewness and kurtosis are the third and fourth order moments, respectively. The third moment measures the asymmetry of the data with respect to a Gaussian distribution and the fourth moment is a measure of the peakedness or flatness of the pdf. (Note that these well-known moments are all of integer order.)

The above are "one-point" statistics, i.e., that take no account of the ordering of the data. In time-series analysis we are also interested in the correlations between different points, recurrent structures or patterns in the datastream. As a first approach to the characterization of two-point correlations, we can compute the auto-correlation function; equivalently (Wiener-Khintchine theorem), we can calculate the power spectral density (PSD). Being based on the squared Fourier amplitudes, the PSD is a second order statistic.

In order to compare "observed, retrieved or simulated" datasets, Davis *et al* (1994) describe a method of data characterization that focuses on the degrees of stationarity and of intermittency for geophysical systems, allowing for arbitrary deviations from Gaussian behavior. Due to their strong nonlinear dynamics, geophysical systems generically exhibit variability over a large range of scales. For statistics parameterized by scale, we expect power law behavior or "scale-invariance". One such statistic is the PSD, E(k), with wavenumber, k (or frequency), playing the role of the scale parameter:

$$E(k) \sim k^{-\beta} \qquad (1)$$

where β is the spectral exponent. For $\beta < 1$ "stationarity" prevails and non-stationarity prevails for $\beta > 1$. The statistical properties of a stationary process are invariant under translation in space or time. If $\beta < 3$, then the data has stationary increments, or more precisely, the statistics of the quantity $[\Sigma(\varphi(x+r) - \varphi(x))]$ will be independent of x for any r within the scaling range.

Non-stationary processes with stationary increments can be investigated using the average of the difference between two datapoints, $\varphi(x+r), \varphi(x)$, x=1,...,L-r, (L is the length of the dataset) over a scale (or lag) r (r=1,2,4,8,...,512), raised to a power q. This can be equated to the scale factor r divided by L, raised to a power $\zeta(q)$ for $q \geq 0$ (q does not have to be an integer). Summarizing, we expect

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$$< |\Delta\varphi(r)|^q > = < |\varphi(x+r) - \varphi(x)|^q > \sim (\tfrac{r}{L})^{\zeta(q)} \qquad q \geq 0, \tag{2}$$

where the left side of equation 2 is the structure function and the structure function exponents, $\zeta(q)$ are to be determined for a range of q's over a variety of scales, r, not to exceed the length of the dataset.

SYSTEM DESCRIPTION

The NASA/Goddard Space Flight Center Raman Lidar (hereafter referred to as the GSFC Lidar) uses an excimer laser with XeF gas to transmit light at 351 nm. The backscattered radiation is collected by a 76 cm diameter Dall-Kirkham telescope. The light backscattered from the laser, as well as the Raman shifted oxygen (373 nm), nitrogen (383 nm) and water vapor (403 nm) wavelengths, are collected. Water vapor mixing ratio profiles are computed using the ratio of the Raman shifted water vapor return signal to the Raman shifted nitrogen return signal. With a differential transmission correction between water vapor and nitrogen wavelengths and a system calibration to correct for the overlap function, then the water vapor to nitrogen ratio is proportional to the water vapor mixing ratio of the atmosphere which are then calibrated with a radiosonde. Water vapor mixing ratio profiles are collected once a minute.

The data used were collected at the Coffeyville, Kansas airport (37.10° N, 95.57° W) as part of the FIRE-II (First ISCCP Regional Experiment - II) and SPECTRE (Spectral Radiance Experiment) simultaneous campaigns from November 12, 1991 to December 7, 1991. Fourteen nights of data were collected, although only four nights were used for this abstract.

RESULTS AND DISCUSSION

The four nights of data used in this analysis are: 11/22/91, 11/26/91, 12/5/91 and 12/6/91. The data at 1 km were retrieved from each profile for each night. These four datasets were used as an ensemble for the structure function analysis. One of the four time-series is plotted in figure 1.

Figure 2 shows the PSD for all four days (open symbols) and for the average of all four (bold circles) on a log-log plot. The data show power law behavior , a straight line, which clearly demonstrates scale-invariance. Furthermore, since $\beta = 1.87$ the data meets the above criteria for non-stationarity and stationary increments.

The structure function data (see eq. 2) for a range of scales r are plotted in figure 3 on a log-log plot, with a least squares fitted line. The $\zeta(q)$ exponents are the slopes of the straight lines (L is constant). This shows scale-invariance (power law behavior) for structure functions of order 1 through 5. The spectral exponent β can, in principle, be retrieved from the second order (q=2) structure function exponent (Monin and Yaglom, 1975):

$$\beta = \zeta(2) + 1. \tag{3}$$

Here we have $\zeta(q) = 0.96$, hence $\beta = 1.96$, which compares well with the PSD estimate of 1.87. This shows that the structure function data are consistent with our previous analyses.

Figure 4 shows the structure exponents $\zeta(q)$ versus the order of the moment q. The straight line plotted for reference, $\zeta(q)$=q/2, corresponds to standard Brownian motion. Deviations from this line is a signature of "multi-fractality" in the system, meaning that in infinite number of exponents are required to characterize the signal. An in-depth structure function exponent analysis with a larger dataset will be discussed during the presentation.

REFERENCES

Davis, Anthony, Alexander Marshak, Warren Wiscombe and Robert Cahalan, 1994: "Multifractal Characterizations of Non-stationary and Intermittency in Geophysical Fields, Observed, Retrieved or Simulated", accepted for publication by *JGR*.

Monin, A.S. and A.M. Yaglom, Statistical Fluid Mechanics, vol. 2, 1975, MIT Press, Boston, Mass., p 683.

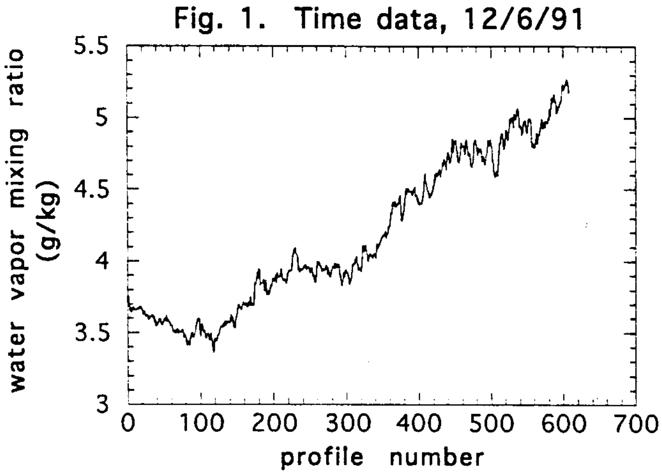

Figure 1. Data at 1 km from each profile taken on 12/6/91 at Coffeyville, Kansas. The time between each profile is one minute.

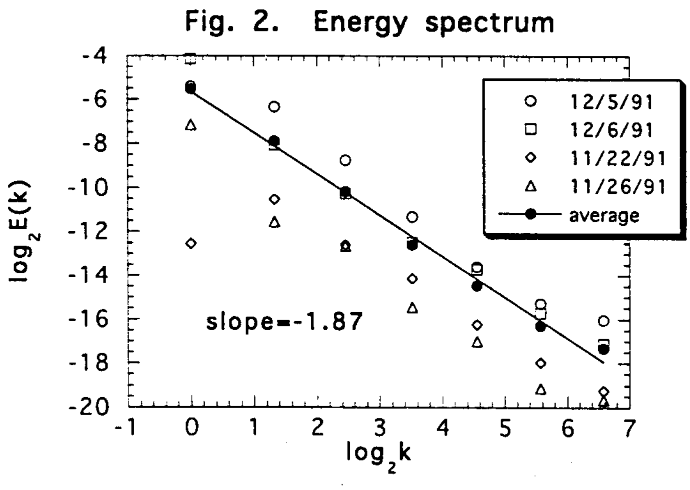

Figure 2. Energy spectra of the data for each night. The log of the Fourier amplitude (g/kg)**2 is plotted versus the log of the wavenumber k=1/r, r=1,2,...128. After FFT performed, data were binned by k octave.

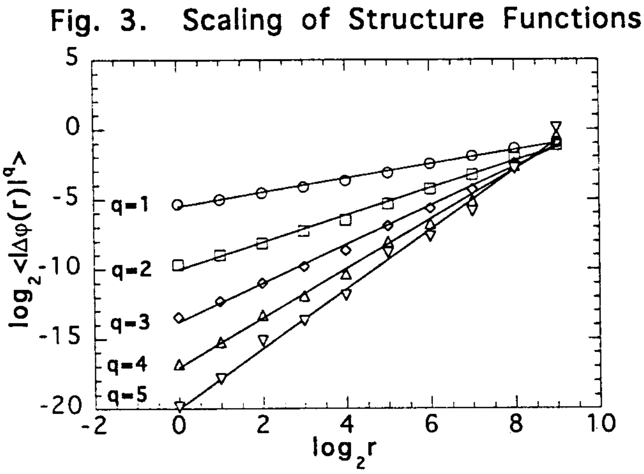

Figure 3. The log of the structure function for various q's is versus the log of the scale parameter. (L is a constant, and doesn't change the slope of the line). The slopes of the fitted lines are the structure function exponents, $\zeta(q)$.

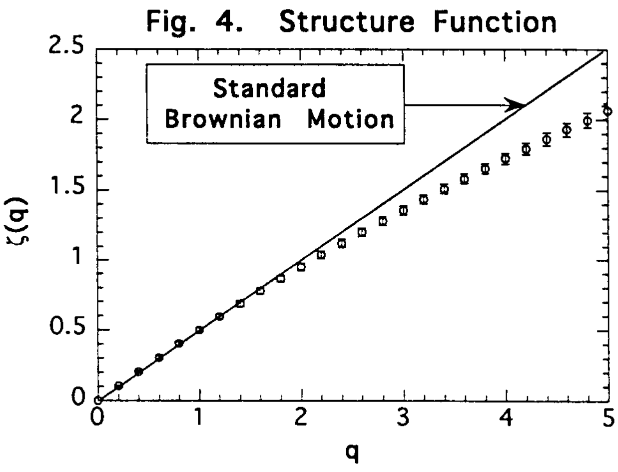

Figure 4. Structure function for various q (not all integers) is plotted along with the structure function fore standard Brownian motion (sBm). Deviations from sBm indicate multi-fractality.