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ON THE INFLUENCE OF THE PULSE-SHAPE UNCERTAINTY ON THE ACCURACY OF FOURIER-DECONVOLUTED LIDAR PROFILES

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INTRODUCTION

The lidar return signal $F(t)$ at the moment t after the pulse emission is described by the convolution

$$(1)\quad F(t) = \int_{-\infty}^{\infty} S(t - 2z'/c)\Phi(z')dz'$$

of the normalized pulse shape $S(\vartheta)$, and the maximum-resolved (short-pulse) lidar profile $\Phi(z')$, where c is the speed of light, and z' is the coordinate along the line of sight; $S(\vartheta) = \mathcal{P}(\vartheta)/\mathcal{P}_p, \mathcal{P}(\vartheta)$ is the pulse power shape and $\mathcal{P}_p$ is its peak value; $t = 2z/c$, z is the distance of sensing with respect to the pulse front. Eq.(1) shows that in the case of long sensing laser pulses (e.g. emitted by a TE(TEA)CO_2 laser) the lidar resolution will be lowered. To improve the lidar resolution, we have developed earlier [1] several deconvolution techniques, one of which is based on Fourier transformation. A question arising here is how the pulse shape uncertainties, caused by unaccurate determination of the pulse shape, influence the accuracy of retrieving $\Phi(z)$ by deconvolution of Eq.(1). An initial analysis of such a problem, based on theoretical estimates and computer simulations, is conducted in the works [1,2]. In the present report we would like to discuss a general feature of the retrieval error caused by the pulse-shape uncertainty in the case of Fourier deconvolution. Namely, the magnitude of the error is on the average equal to the ratio of the algebraic pulse-uncertainty area to the true-pulse area.

THEORETICAL NOTES

The Fourier deconvolution of Eq.(1) leads to the algorithm

$$(2)\quad \Phi(z \equiv ct/2) = (\pi c)^{-1} \int_{-\infty}^{\infty} \left[\tilde{F}(\omega)/\tilde{S}(\omega)\right] \exp(-j\omega t)\, dt,$$

where $\tilde{S}(\omega) = \int_{-\infty}^{\infty} S(\vartheta) \exp(j\omega\vartheta)\, d\vartheta$ and $\tilde{F}(\omega) = \int_{-\infty}^{\infty} F(t) \exp(j\omega t)\, dt$ are Fourier transforms of $S(\vartheta)$ and $F(t)$, respectively, j is imaginary unity; $\tilde{\Phi}(\omega) \equiv \tilde{F}(\omega)/\tilde{S}(\omega) = (c/2)\int_{-\infty}^{\infty} \Phi(t) \exp(j\omega t)dt$. If we represent the measured pulse shape $S_m(\vartheta)$ as a sum $S_m(\vartheta) = S(\vartheta) + f(\vartheta)$ of the true pulse shape $S(\vartheta)$ and a deterministic or random uncertainty $f(\vartheta)$ we will obtain from Eq.(2) the retrieval error δ_Φ in the form

$$(3)\quad \delta_\Phi(z = ct/2) = \Phi_r(z) - \Phi(z) = -(\pi c)^{-1} \int_{-\infty}^{\infty} \frac{\tilde{\Phi}(\omega)\tilde{f}(\omega)}{\tilde{S}(\omega) + \tilde{f}(\omega)} \exp(-j\omega t)d\omega,$$

where $\Phi_r(z)$ is the restored lidar profile using $S_m(\vartheta)$, and $\tilde{f}(\omega) = \int_{-\infty}^{\infty} f(\vartheta) \exp(j\omega\vartheta)d\vartheta$ is the Fourier transform of $f(\vartheta)$. As far as $\tilde{\Phi}(\omega), \tilde{S}(\omega)$ and $\tilde{f}(\omega)$ are integrals over all values of $z = ct/2$, Eq.(3) does not represent, in general, a local dependence of $\delta_\Phi(z)$ on $\Phi(z)$ and $f(z)$.

If $f(\vartheta)$ is a slowly varying prolonged deterministic function as compared to $S(\vartheta)$, Eq.(3) may be simplified to the form

$$(4)\quad \delta_\Phi(z = ct/2) \approx -p_{\text{eff}}^{-1} \int_{-\infty}^{\infty} \Phi(z') f(t - 2z'/c)dz' = -\left(p_{\text{un}}/p_{\text{eff}}\right) \bar{\Phi}(t)$$

indicating a proportionality to the magnitudes of Φ and f, and involving interactions of f with earlier ($at\ t' < t$) values of Φ. In Eq.(4), $p_{\text{eff}} = (c/2)\int_{-\infty}^{\infty} S(\vartheta)\, d\vartheta$ and $p_{\text{un}} = (c/2)\int_{-\infty}^{\infty} f(\vartheta)\, d\vartheta$ are the pulse- shape area and the uncertainty area, respectively; $\bar{\Phi} = p_{\text{un}}^{-1}(f * \Phi)$ is, in fact, a weighted, by the uncertainty, average of Φ; $*$ denotes a convolution. The right-hand part of Eq.(4) is usefull only when $f(\vartheta)$ does not change its sign. Then we see that the error δ_Φ (as well as the relative error $\delta_\Phi/\bar{\Phi}$) is proportional to $p_{\text{un}}/p_{\text{eff}}$ i.e. to the potential relative contribution of the uncertainty to $F(t)$ (compared to the true-pulse contribution).

The cut of a pulse spike is a typical case of a fast varying short-range uncertainty. In this case, on the basis of Eqs.(1) and (2) we obtain

(5a) $$\bar{\delta}_\Phi(t)/\Phi(t) \approx p_{\text{S}}/p_{\text{R}}$$

(5b) $$\bar{\delta}_\Phi(t)/\Phi(t) \approx p_{\text{S}}/p_{\text{eff}}$$

as far as $p_{\text{S}} \ll p_{\text{eff}}$. In Eqs.(5), $p_{\text{S}} = (c/2)\int_{-\infty}^{\infty} f_{\text{S}}(t)\, dt$ is the cut-spike area, and $f_{\text{S}}(t)$ is a function describing the cut spike; $\bar{\delta}_\Phi(t) = p_{\text{R}}^{-1}(f_{\text{R}} * \delta_\Phi) \approx p_{\text{eff}}^{-1}(S * \delta_\Phi)$ is interpreted as a weighted, by the remaining tail, average of δ_Φ, f_{R} describes the remaining tail ($S = f_{\text{S}} + f_{\text{R}}$), and p_{R} is its area ($p_{\text{R}} = p_{\text{eff}} - p_{\text{S}}$). So, Eqs.(5) show that, as above, the relative (with respect to the whole true pulse) contribution of the spike to $F(t)$ determines on the average the magnitude and the range of the retrieval errors.

The results obtained above suggest that in the case of sinusoidal deterministic uncertainties or random uncertainties with, respectively, period T or correlation time τ_c less than the least period of varying $\Phi(z)$, the retrieval error will be proportional to T or τ_c, because in such cases the effect of the uncertainty on $F(t)$ will be averaged and will vanish with the decrease of T or τ_c. Then the algebraic area of the uncertainty decreases.

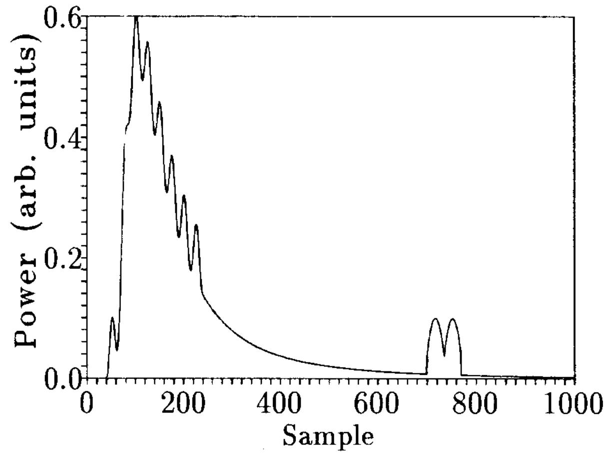

Figure 1: Model of the maximum-resolved profile $\Phi(z)$ versus sample number ($1 sample = 15m$).

SIMULATIONS

The models of $\Phi(z)$ and $S(t)$ used in the simulations are given in Figs.1 and 2, respectively.

Three models of uncertainties are simulated, namely:

i) a prolonged smooth uncertainty given by the expression

(6a) $f(\vartheta) = qS(\vartheta), |q| < 1$;

ii) a spike cut represented in Fig.2 by a dashed curve;

iii) a random uncertainty

(6b) $f(\vartheta) = \sigma_f \tilde{f}(\vartheta)$

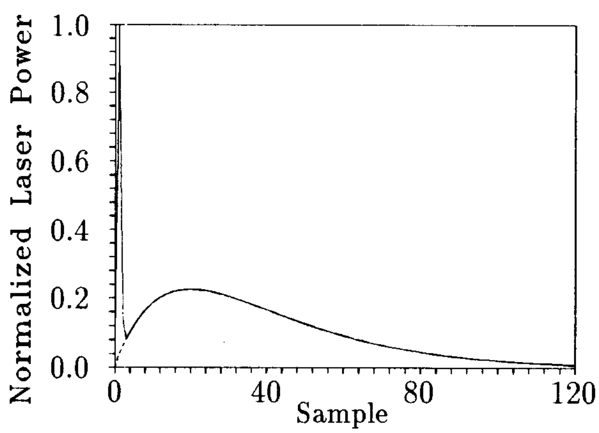

Figure 2: Graph of the normalized true-pulse shape $S(\vartheta)$.

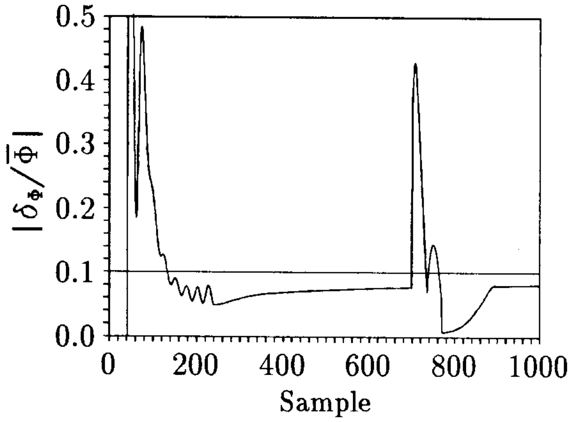

Figure 3: Module of the relative error $\delta_\Phi/\bar{\Phi}$ in the case of a prolonged smooth uncertainty with $q = 0.1$.

with standart deviation $\sigma_f = qS(\vartheta)$ $(0 < q < 1)$, where the factor $\tilde{f}(\vartheta)$ is a stationary Gaussian-distributed and Gaussian-correlated zero-mean random function with variance $D\tilde{f} = 1$ and correlation time τ_c.

The lidar return signal $F(t)$ is simulated on the basis of Eq.(1) using the true pulse shape $S(\vartheta)$. The deconvolution is performed by use of the measured pulse shape $S_m(\vartheta)$, and the obtained errors are processed in order to test the theoretical results and suppositions in the previous Section 2.

In Fig.3, the module of the relative error $\delta_\Phi/\bar{\Phi}$, in the case of a prolonged smooth uncertainty, is compared with the level $p_{\rm un}/p_{\rm eff}$. It is seen that $|\delta_\Phi/\bar{\Phi}|$ oscillates around a level which is a little lower than $p_{\rm un}/p_{\rm eff}$.

For the case of the spike cut in Fig.2, the relative error $\bar{\delta}_\Phi/\Phi$ oscillates around the value of $p_{\rm S}/p_{\rm R}$ (Fig.4).

The results from the simulations of random uncertainties show that the error δ_Φ depends, as can be expected, on the noise level $(\sim q)$. Another interesting question, being investigated now, is in what way δ_Φ depends on the correlation time τ_c.

CONCLUSION

Thus, the analysis and the simulations conducted above show that the retrieval error caused by the pulse-shape uncertainty is proportional on the average to the real or

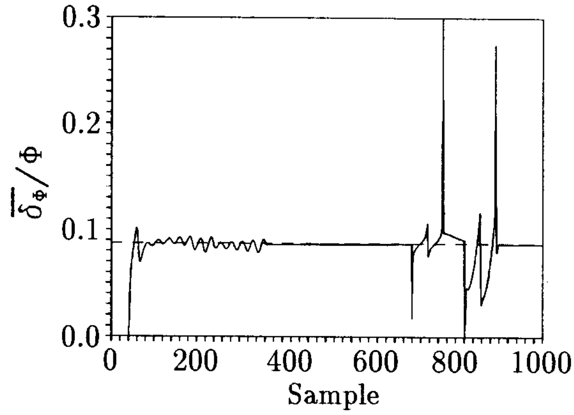

Figure 4: Relative error $\bar{\delta}_\Phi/\Phi$ for the case of a spike cut.

the potential contribution of the uncertainty to the lidar return signal. The relative error is proportional to the relative contribution, with respect to the true pulse contribution, characterized by the ratio of the uncertainty area to the true pulse area.

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