

G.G. Matvienko, G.Ya. Patrushev, A.L. Afanas'ev,  
A.I. Grishin, A.P. Rostov, Yu.M. Vorevodin  
Institute of Atmospheric Optics  
SB Russian Academy of Sciences  
1, Akademicheskii Ave., Tomsk,  
634055, Russian Federation  
Tel. (8-382-2) 25-84-96, Fax (8-382-2) 25-90-86  
E-mail: zuev@iao.tomsk.su

The urgency of development of laser technique for sounding of wind parameters is dictated by a practical demand for operative information on the profiles of wind velocity and direction in the lower atmosphere as well as the wind velocity fluctuations. The present paper discusses the results of strict description of spatial-temporal characteristics of signal fluctuations of the incoherent aerosol lidar and the interpretation of lidar sounding data.

If the assumption is made that the components of wind velocity  $V_x, V_y, V_z$  are distributed following the normal law with the average value  $V_0$  and variances  $\sigma^2$ , using the spatial spectrum of aerosol particle concentration fluctuations of the form:

$$\Phi_N(\kappa) = 0,033 \frac{C_N^2}{\kappa}^{-11/3} (1 - e^{-\kappa^2/\kappa_0^2})$$

where  $\kappa_0 = 2\pi/L_0$ ,  $L_0$  is the outer scale of turbulence,  $C_N^2$  is the structural characteristics of particle concentration fluctuations, according to [1] the analytical expression for the normalized spatial-temporal correlation function of backscattering is:

$$\frac{B(\vec{r}, \tau)}{B(0,0)} = B^{-1} \left\{ \left[ \left( a_v^2 + \kappa_0^{-2} + \frac{1}{2} \sigma^2 \tau^2 \right)^{1/3} {}_1F_1 \left[ -\frac{1}{3}, \frac{3}{2}, \frac{-\left( \vec{r} - V_0 \tau \right)^2}{4 \left( a_v^2 + \kappa_0^{-2} + \frac{1}{2} \sigma^2 \tau^2 \right)} \right] - \left[ \left( a_v^2 + \frac{1}{2} \sigma^2 \tau^2 \right)^{1/3} {}_1F_1 \left[ -\frac{1}{3}, \frac{3}{2}, \frac{\left( \vec{r} - V_0 \tau \right)^2}{4 \left( a_v^2 + \frac{1}{2} \sigma^2 \tau^2 \right)} \right] \right\}, \quad (1)$$

where  $B = (a_v^2 + \kappa_0^{-2})^{1/3} - a_v^{2/3}$ ,  $a_v$  is dimension of scattering volumes;  $\vec{r}$  is the volume spacing,  $\tau$  is the time delay;  ${}_1F_1(a, b, x)$  is the hypergeometric function. This expression was obtained assuming the equality of dispersions of the wind velocity components. However, in practice, the presence of anisotropy of wind velocity fluctuations is often observed. To estimate the effect of this factor the analytical calculation of  $B(\vec{r}, \tau)/B(0,0)$  is carried out for the case when the vertical fluctuations of wind velocity are different from fluctuations in a horizontal plane ( $\sigma_x = \sigma_y = \sigma \neq \sigma_z$ ). The expression obtained is of the form:

$$\frac{B(\vec{r}, \tau)}{B(0,0)} = B^{-1} \left\{ \frac{\left[ \left( a_v^2 + \kappa_0^{-2} + \frac{1}{2} \sigma^2 \tau^2 \right)^{5/6} \right]}{\left[ \left( a_v^2 + \kappa_0^{-2} + \frac{1}{2} \sigma_z^2 \tau^2 \right)^{1/2} \right]} \times \right.$$

$$\times \Theta_1 \left[ \frac{1}{2}, -\frac{1}{3}, \frac{11}{6}, \frac{3}{2}, \frac{-\frac{1}{2}(\sigma^2 - \sigma_z^2)\tau^2}{a_v^2 + \kappa_0^{-2} + \frac{1}{2}\sigma_z^2\tau^2}; \frac{-(r-V_0\tau)^2}{4(a_v^2 + \kappa_0^{-2} + \frac{1}{2}\sigma_z^2\tau^2)} \right] - \frac{(a_v^2 + \frac{1}{2}\sigma_z^2\tau^2)^{5/6}}{(a_v^2 + \frac{1}{2}\sigma_z^2\tau^2)^{1/2}} \times$$

$$\times \Theta_1 \left[ \frac{1}{2}, -\frac{1}{3}, \frac{11}{6}, \frac{3}{2}, \frac{-\frac{1}{2}(\sigma^2 - \sigma_z^2)\tau^2}{a_v^2 + \frac{1}{2}\sigma_z^2\tau^2}; \frac{-(r-V_0\tau)^2}{4(a_v^2 + \frac{1}{2}\sigma_z^2\tau^2)} \right] \quad (2)$$

where  $\Theta_1(a, a', b; c; t, x)$  is the hypergeometric function of the two variables [2]. It should be noted that if in Eq. (2) one assumes  $\sigma_z = \sigma$ , then  $t=0$ ,  $\Theta_1(a, a', b; c; 0, x) = {}_1F_1(a', c, x)$  and Eq. (2) coincides with Eq. (1).

The behavior of  $B(r, \tau)/B(0, 0)$  at  $\sigma = 0$  and  $\sigma_z = \sigma$  calculated by the formula (2) for the values  $a_v = 1\text{m}$ ,  $L_0 = 80\text{m}$ ,  $r = 5\text{m}$ ,  $V_0 = 1\text{m/s}$  is given in Fig. 1. It is clear that the anisotropy of wind velocity fluctuations affect the cross-correlation function and leads to the decrease of variance of the vertical component of wind velocity  $\sigma_z^2$ , as compared to the horizontal one  $\sigma^2$ , results in the shift of the correlation maximum in the region of large values of  $\tau$ , produces the broadening of correlation function and the increase of maximum correlation level as compared to the case of isotropic fluctuations.

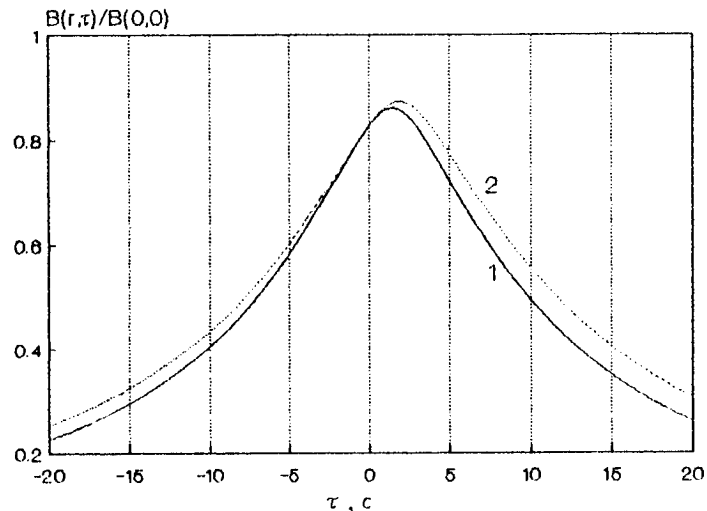


Fig. 1. The influence of anisotropy of wind velocity fluctuations on the cross-correlation function of lidar returns  
 1 -  $\sigma_x = \sigma_y = \sigma_z = \sigma$ ;  
 2 -  $\sigma_x = \sigma_y = \sigma$ ,  $\sigma_z = 0$

Another method of lidar return signals processing is based on the use of methods of coherent analysis. In [1] the effect of wind fluctuations on spectral characteristics of the lidar returns was taken into account for the case  $\sigma^2/V^2 \ll 1$ . However, in a series of meteorological situations the ratio  $\sigma^2/V^2$  may be of the order of 1 or more. An account of wind fluctuations at an arbitrary ratio  $\sigma^2/V^2$  for a mutual spectrum

$$W(r, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} B(r, \tau) e^{-i\omega\tau} dt$$

results in the expression

$$W(r, \omega) = c \frac{1}{V_0} \exp\left[\frac{-i\omega r}{V_0}\right] \int_0^\infty \kappa \Phi_N(\kappa) \exp\left[-a_v^2 \kappa^2 - \frac{1}{2} \frac{\sigma_v^2 r^2}{V_0^2} \kappa^2\right] \times$$

$$\times \left\{ \operatorname{erf}\left[\frac{V_0}{\sqrt{2}\sigma} - \frac{\omega}{\sqrt{2}\kappa} + i \frac{\sigma r \kappa}{\sqrt{2}V_0}\right] + \operatorname{erf}\left[\frac{V_0}{\sqrt{2}\sigma} + \frac{\omega}{\sqrt{2}\kappa} - i \frac{\sigma r \kappa}{\sqrt{2}V_0}\right] \right\} d\kappa, \quad (3)$$

where  $\omega = 2\pi f$ ,  $f$  is the frequency. (Here, for simplicity, the spacing of scattering volumes  $r$  is oriented along the direction of mean velocity  $V_0$  and  $\sigma_x = \sigma_y = \sigma_z = \sigma$ ).

The behavior of the coherence spectrum  $\gamma(r, \omega) = \frac{|W(r, \omega)|}{|W(0, 0)|}$  and the phase spectrum  $\varphi(r, \omega) = \arctg \frac{\operatorname{Im} W(r, \omega)}{\operatorname{Re} W(r, \omega)}$

is presented in Fig.2. The given theoretical curves are obtained by numerical integration (3) for the values  $a_v = 0,2\text{m}$ ,  $L_0 = 80\text{m}$ ,  $r = 5\text{m}$ . The curves

1-3 describe the decrease of  $\gamma^2(r, \omega)$  due to the wind velocity fluctuations. In the low frequency region the coherence spectrum is saturated for the level depending on  $\sigma_v^2 = 3\sigma^2$  and in the high frequency region it tends to zero. It should be noted that in the

absence of wind fluctuations the coherence spectrum does not depend on frequency and equals 1 and the phase spectrum is linear, e.g., curves 4' and 5'.

The presence of wind fluctuations leads to the increase of slope of the phase spectrum in proportion to

$\sigma_v$ , e.g., curves 1'-3'. In this case, for larger ratios  $\sigma_v/V_0$  (curve 2') a marked nonlinear behavior of  $\varphi(r, \omega)$  is observed.

**Fig.2** The behavior of coherence spectra and the phase in the presence of strong fluctuations of wind velocity (theoretical calculation): 1-3 are the coherence spectra; 1'-5' are the phase spectra;

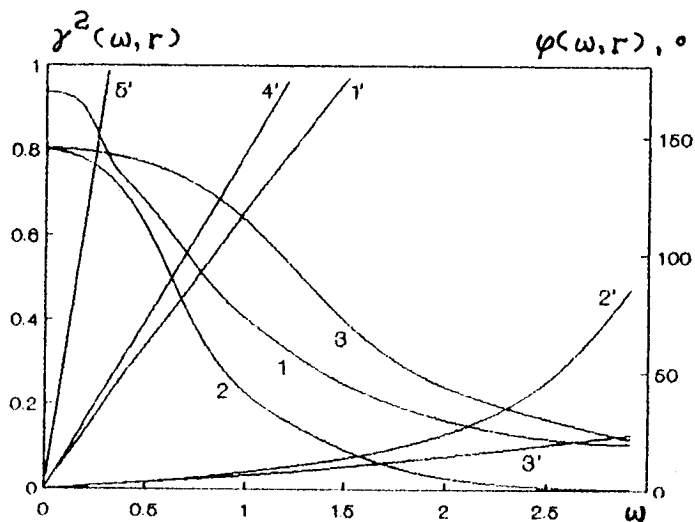
a)  $V_0 = 2\text{m/s}$ ; 4' -  $\sigma_v/V_0 = 0$ ;

1, 1' -  $\sigma_v/V_0 = 0,5$ ;

3, 3' -  $\sigma_v/V_0 = 5$ ;

b)  $V_0 = 0,5\text{m/s}$ ; 5' -  $\sigma_v/V_0 = 0$ ;

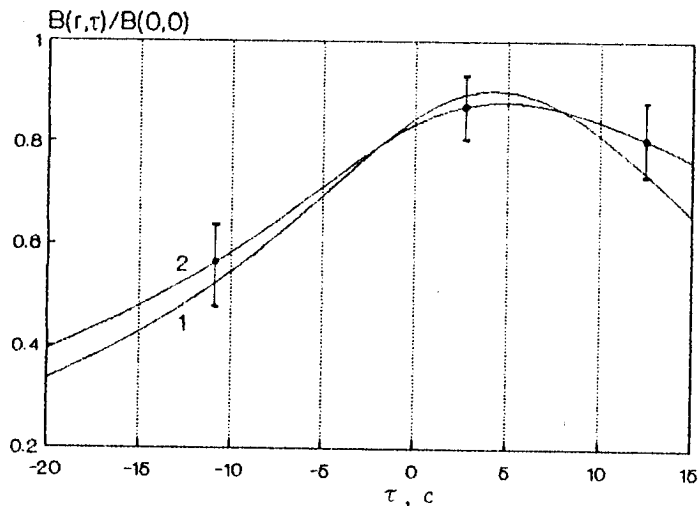
2, 2' -  $\sigma_v/V_0 = 10$ .



For experimental check of theoretical calculations the field experiments were carried out using a double-path lidar [3] and a three-component ultrasound anemometer [4] providing measurement of mean values and fluctuations of wind velocity. The lidar operating wavelength was  $0,53 \mu\text{m}$ , laser beams were transmitted to the atmosphere in horizontal direction at an angle of  $3,5^\circ$ . Temporal series of lidar returns recorded from the scattering volumes located close to the anemometer were subjected to correlation processing. Figure 3 gives a comparison of calculated and experimental correlation functions obtained at the parameter values  $r = 7,5\text{m}$ ,  $a_x = 1,5$ ,

$a_y = a_z = 0,2\text{m}$ ;  $\sigma_x = \sigma_y = 0,22\text{ m/s}$ ;  $\sigma_z = 0,077\text{m/s}$ ;  $V_0 = 1,41\text{ m/s}$ ;  $L_0 = 80\text{m}$ .

Fig. 3. Experimental (1) and theoretical (2) correlation functions of lidar returns at 2,5m height. The angle between  $V_0$  and  $r$  equals  $20^\circ$ . Vertical sections indicate the root-mean-square deviations.



The data given in Fig.3 show the satisfactory agreement, the curves coincide both in the shape of functions and in the correlation values. Good agreement between the theoretical analysis and the experiment confirms the validity of theoretical approach and the necessity of account of strong fluctuations of wind velocity and anisotropy of its fluctuations.

Different types of remote sensing of the fluctuating wind velocity are proposed on the basis of the compared results.

The authors thank I.A.Razenzov for giving a help in the experiment.

#### References

1. Yu. S. Balin, M. S. Belen'kii, I. A. Razenzov, N. V. Safonova // Atmospheric Optics, 1988, v. 1, No. 8, pp. 77-83.
2. A. P. Prudnikov, Yu. A. Brychkov, O. N. Marichev // Integrals and Series. Supplementary Chapters. M., Nauka, 1986, 800 pp.
3. G. G. Matvienko // Atmospheric Optics, 1988, v. 1, No. 6, pp. 3-15.
4. A. P. Rostov // Atmospheric and Oceanic Optics, 1993, v. 6, No. 1, pp. 102-106.