LIDAR EQUATION WITH THE JOINT ACCOUNT OF THE SMALL-ANGLE MULTIPLE SCATTERING AND THE SINGLE ANISOTROPIC SCATTERING AT LARGE SCATTERING ANGLES

Victor V.Veretennikov

Institute of Atmospheric Optics of the SB RAS
1, Akademicheskii Prospect, Tomsk, 634055, Russia
E-mail: vvv@iao.ru

ABSTRACT

A new approach is suggested in the paper to the approximate consideration of backscattering anisotropy in lidar returns. It reduces the solution of the problem to the calculation of irradiance distribution in the medium. The solution has been obtained with the use of the Green's function method and the optical reciprocity theorem. The principal assumption used in the derivation of the new lidar equation consisted in the δ- approximation of the Green's function in terms of the angular coordinate. At the same time the information about the scattering phase function both in the small-angle region, and in the vicinity of backscattering region is completely kept. The obtained expressions were applied to assess the effect of the backscattering anisotropy in the calculation of the contribution from multiple scattering in sensing of the liquid droplet cloud and the seawater.

1. FORMULATION OF THE PROBLEM

A considerable progress has been made by now in construction of numerical-analytical models describing the lidar signal with the allowance made for multiple scattering. Simplifications usually used for this purpose account for the small-angle multiple scattering and only one event of scattering to a large angle [1-3].

In solving the problems of radiative transfer through the media with anisotropic scattering in the coefficient of directed light scattering $\beta(\gamma)=\sigma x(\gamma)$ it is a usual practice to separate the forward peak of $\beta_1(\gamma)=\sigma_1 x_1(\gamma)$, which describes the scattering at small angles. Designating the residual part as $\beta_2(\gamma)=\beta(\gamma)-\beta_1(\gamma)$, we can write the following equation for the normalized scattering phase function:

$$x(\gamma)=a_1x_1(\gamma)+a_2x_2(\gamma) \tag{1}$$

with the weighting coefficients $a_1=\sigma_1/\sigma$ and $a_2=\sigma_2/\sigma$, $\sigma=\sigma_1+\sigma_2$, where σ is the scattering coefficient. In this case account of only one event of scattering at the large angle results in the solution of the following non-stationary problem of the radiative transfer theory [4]

$$DI=L_1I+Q, \tag{2}$$

where

$$D\equiv\frac{1}{c}\frac{\partial}{\partial t}+(\mathbf{n}\nabla)+\varepsilon,$$

$$L_1I\equiv\sigma_1\int_{4\pi}I(\mathbf{R},\mathbf{n}')x_1(\mathbf{n},\mathbf{n}')d\mathbf{n}', \tag{3}$$

$$Q(\mathbf{R},\mathbf{n})=\sigma_2\int_{4\pi}I_1(\mathbf{R},\mathbf{n}')x_2(\mathbf{n},\mathbf{n}')d\mathbf{n}', \tag{4}$$

$\mathbf{R}=(x,y,z)$ is the radius-vector of a point; $\mathbf{n}$ is the unit direction vector; Q is the source density function; ε is the extinction coefficient; $I_1(\mathbf{R},\mathbf{n}')$ is the spatial-angular intensity distribution of the radiation at the forward propagation of the sounding pulse. The function $I_1(\mathbf{R},\mathbf{n}')$ satisfies Eq. 2 under the condition $Q=0$. Within the framework of the small-angle approximation, the intensity $I_1(\mathbf{R},\mathbf{n}')$ has a significant value only in the small vicinity of the initial direction of the light beam propagation. Let this direction coincide with the direction of the axis Oz. We assume that the angle $\gamma=(\mathbf{n}\wedge\mathbf{n}')$ between the directions $\mathbf{n}$ and $\mathbf{n}'$ is close to π and, consequently, the angle $\theta=\pi-\gamma$ is small. On these assumptions, the right-hand side of Eq. 4 can be presented as a double integral

$$Q(\mathbf{R},\mathbf{n}_\perp)=\sigma_2\iint x_\pi(|\mathbf{n}_\perp+\mathbf{n}'_\perp|)I_1(\mathbf{R},\mathbf{n}'_\perp)d\mathbf{n}'_\perp, \tag{5}$$

where $\mathbf{n}_\perp$ n and $\mathbf{n}'_\perp$ are the projections of the vectors $\mathbf{n}$ and $\mathbf{n}'$ onto the plane, orthogonal to the axis Oz, $x_\pi(\theta)=x_2(\pi-\theta)$. It is necessary to set boundary conditions for the solution of Eq. 2. For simplicity, we will consider below the problem for the case that the medium is irradiated with a point source of a pulsed unidirectional (PUD) radiation, which generates the following intensity distribution on the medium boundary $z=0$

$$I(\mathbf{r}_\perp,z=0,\mathbf{n}_\perp,t)=\delta(\mathbf{n}_\perp-\mathbf{n}_{0\perp})\delta(\mathbf{r}_\perp-\mathbf{r}_{0\perp})\delta(t) \tag{6}$$

at $\mathbf{n}_{0\perp}=0,\quad \mathbf{r}_{0\perp}=0$.

2. SOLUTION METHOD

We use the method of Green's function and the optical reciprocity theorem for the solution of Eq. 2 with the boundary condition (6). The intensity $I_1(\mathbf{r}_\perp, z, \mathbf{n}'_\perp, t)$ in Eq. 5 can be presented as

$$\begin{aligned} &I_1(\mathbf{r}_\perp, z, \mathbf{n}'_\perp, t) = \\ &= G(\mathbf{r}_{0\perp}, z=0, \mathbf{n}_{0\perp} \to \mathbf{r}_\perp, z, \mathbf{n}'_\perp)\delta(t - z/c), \end{aligned} \tag{7}$$

where $G(\mathbf{r}_{0\perp}, z=0, \mathbf{n}_{0\perp} \to \mathbf{r}_\perp, z, \mathbf{n}'_\perp)$ is the small-angle Green's function of the stationary problem. The function $G(\mathbf{r}_{0\perp}, z=0, \mathbf{n}_{0\perp} \to \mathbf{r}_\perp, z, \mathbf{n}'_\perp)$ has a sharp peak in the direction $\mathbf{n}'_\perp = \mathbf{n}_{0\perp}$ of the light beam irradiating the medium and decreases fast as the angle $\gamma' = |\mathbf{n}'_\perp|$ deviates from zero. The scattering phase function $x_\pi(|\mathbf{n}_\perp + \mathbf{n}'_\perp|)$ to the contrary, changes much more slowly. Therefore, we can expect that substitution of the small-angle Green's function by the two-dimensional δ-function

$$\begin{aligned} &G(\mathbf{r}_{0\perp}, z=0, \mathbf{n}_{0\perp} \to \mathbf{r}_\perp, z, \mathbf{n}'_\perp) = \\ &= E_1(\mathbf{r}_\perp)\delta(\mathbf{n}'_\perp - \mathbf{n}_{0\perp}) \end{aligned} \tag{8}$$

does not lead to a large error in the integral (5). The normalizing factor

$$E_1(\mathbf{r}_\perp) = \iint G(\mathbf{r}_{0\perp}, z=0, \mathbf{n}_{0\perp} \to \mathbf{r}_\perp, z, \mathbf{n}'_\perp) d\mathbf{n}'_\perp \tag{9}$$

in Eq. 8 is the spatial irradiance, generated by the PUD source (6) at the point $\mathbf{r}_\perp$. Taking into account the approximation (8), we obtain the following equation for the source function of the stationary problem:

$$Q(\mathbf{R}, \mathbf{n}_\perp) = \sigma_2 x_\pi(|\mathbf{n}_\perp|) E_1(\mathbf{r}_\perp). \tag{10}$$

Assume then that the observations in lidar measurements are conducted in the plane $z = 0$. Then, using the Green's function method and the optical reciprocity theorem [5], we can write the following equation for the light field in the observation plane, corresponding to the source density $Q(\mathbf{R}, \mathbf{n}_\perp)$:

$$\begin{aligned} &I(\mathbf{r}_\perp, z=0, \mathbf{n}_\perp, t) = (\sigma_2 c/2) \iint_S E_1(\mathbf{r}'_\perp) d\mathbf{r}'_\perp \times \\ &\times \iint x_\pi(|\mathbf{n}'_\perp|) G(\mathbf{r}_\perp, z=0, -\mathbf{n}_\perp \to \mathbf{r}'_\perp, z', -\mathbf{n}'_\perp) d\mathbf{n}'_\perp. \end{aligned} \tag{11}$$

Here S is the integration plane $z' = ct/2$. By analogy with the case considered in Eq. 8, in the integration over the angular variable in Eq. 11, the Green's function is approximated as:

$$G(\mathbf{r}_\perp, z=0, -\mathbf{n}_\perp \to \mathbf{r}'_\perp, z', -\mathbf{n}'_\perp) = E_2 \delta(\mathbf{n}'_\perp - \mathbf{n}_\perp), \tag{12}$$

where

$$\begin{aligned} &E_2(\mathbf{r}_\perp, -\mathbf{n}_\perp \to \mathbf{r}'_\perp, z') = \\ &= \iint G(\mathbf{r}_\perp, z=0, -\mathbf{n}_\perp \to \mathbf{r}'_\perp, z', -\mathbf{n}'_\perp) d\mathbf{n}'_\perp \end{aligned} \tag{13}$$

is the spatial irradiance generated at the point $\mathbf{R} = (\mathbf{r}'_\perp, z')$ by a fictitious PUD radiation from a source located at the point $\mathbf{r}_\perp$ of the plane $z = 0$ and emitting along the direction $-\mathbf{n}_\perp$. This leads to the following equation for the light field intensity distribution in the plane $z = 0$:

$$\begin{aligned} &I(\mathbf{r}_\perp, z=0, \mathbf{n}_\perp, t) = (\sigma_2 c/2) x_\pi(|\mathbf{n}_\perp|) \times \\ &\times \iint_S E_1(\mathbf{r}'_\perp) E_2(\mathbf{r}_\perp, -\mathbf{n}_\perp \to \mathbf{r}'_\perp, z') d\mathbf{r}'_\perp. \end{aligned} \tag{14}$$

In the small-angle approximation, the property of invariance is true for the spatial irradiance:

$$E_2(\mathbf{r}_\perp, \mathbf{n}_\perp \to \mathbf{r}'_\perp, z') = E_0(|\mathbf{r}'_\perp - \mathbf{r}_\perp - z'\mathbf{n}_\perp|), \tag{15}$$

where $E_0(\mathbf{r}_\perp)$ is the beam spread function (BSF), that is, the irradiance distribution in the plane z, generated by the PUD radiation from a source located at the origin of coordinates and emitting along the direction of the axis Oz. Taking into account the property (15), the integral in Eq. (14) takes the form of the two-dimensional convolution in the plane S:

$$\begin{aligned} &I(\mathbf{r}_\perp, z=0, \mathbf{n}_\perp, t) = (\sigma_2 c/2) x_\pi(|\mathbf{n}_\perp|) \times \\ &\times \iint_S E_1(\mathbf{r}'_\perp) E_0(|\mathbf{r}_\perp - z'\mathbf{n}_\perp - \mathbf{r}'_\perp|) d\mathbf{r}'_\perp. \end{aligned} \tag{16}$$

Using Eq. 16, we can easily calculate the power of the lidar return for the given parameters of the receiving system. Let, for example, the sensitivity function of the lidar receiving system has a circular symmetry and the stepwise behavior over the variables $r = |\mathbf{r}_\perp|$ and $\gamma = |\mathbf{n}_\perp|$. Then, if the lidar emits a δ-pulse with the unit energy, the lidar return power detected at the time $t = 2z/c$ is described by the common equation:

$$P(z) = P_1(z)[1 + m(\gamma_r)], \tag{17}$$

where $P_1(z)$ is the lidar return in the single scattering approximation. The correction factor $m(\gamma_r)$ specifies the ratio of multiply and singly scattered components of lidar return and for considered conditions has the form

$$m(\gamma_r) = 2\pi \int_0^{z\gamma_r} \hat{x}(\rho / z)\hat{E}(\rho)\rho d\rho\,, \qquad (18)$$

where

$$\hat{E}(\rho) = \frac{1}{2\pi}\int_0^{\infty} \nu J_0(\rho\nu)[e^{g(\nu)} - 1]d\nu\,, \qquad (19)$$

$$\hat{x}(\gamma) = x_\pi(\gamma) / x_\pi(0)\,, \qquad (20)$$

$$g(\nu) = 2\int_0^{z} \sigma_1(z-s)\tilde{x}_1(\nu s)ds\,. \qquad (21)$$

Eq. 18 is derived under additional conditions: $z > R_r / \gamma_r$ and $x_\pi(\gamma) = x_\pi(0)$, $0 < \gamma \le R_r / z$, R_r and γ_r are the radius of the entrance pupil and the field-of-view (FOV) half-angle of the receiving system. Eqs. 18-21 employ the following designations: ν is the spatial frequency; $J_0(.)$ is the first-kind zero-order Bessel function; $\tilde{x}_1(p)$ is the Hankel transform of the small-angle scattering phase function.

Eqs. 17-21 give full description of the lidar return formed due to multiple scattering at small angles and the single anisotropic scattering, taken into account near the backward direction.

3. RESULTS OF NUMERICAL SIMULATION

In this Section, we consider some examples of the function $m(\gamma_r)$ (18) calculated taking into account the backscattering anisotropy as compared with similar results obtained assuming $x_\pi(\gamma)$ = *const*. The calculations are based on two basic models of scattering media, which describe properties of clouds and sea water. The model type of "*Cloud C1*" [6] is considered, in which the modal radius of particles $\bar{r}_s$ is varied in addition, when optical-microphysical properties of a cloud are specified.

The scattering phase function is shown in Fig. 1 at a wavelength λ = 0.532 µm in the vicinity of a backward direction for cloudy particles with different modal radius. General feature for all curves, given in Fig. 1, is the sharp decrease at moving from the scattering angle 180°, and the presence of a diffraction maximum (glory) in the range of angles 177 - 179°. The location and amplitude of the maximum depends on size of modal radius of particles $\bar{r}_s$.

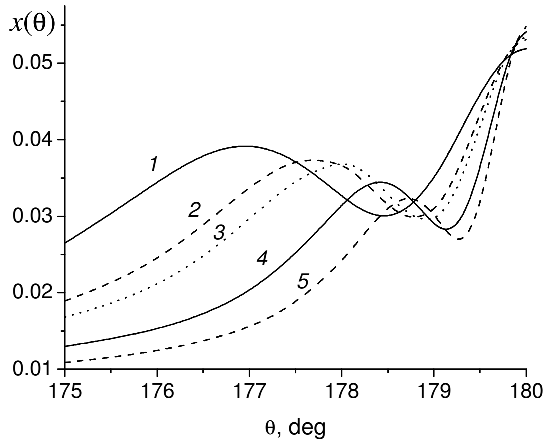

Fig. 1. Model dependences of the scattering phase function for the cloud particles with modal radius $\bar{r}_s$ = 4, 5.33, 6, 8, and 10 µm (curves *1–5*).

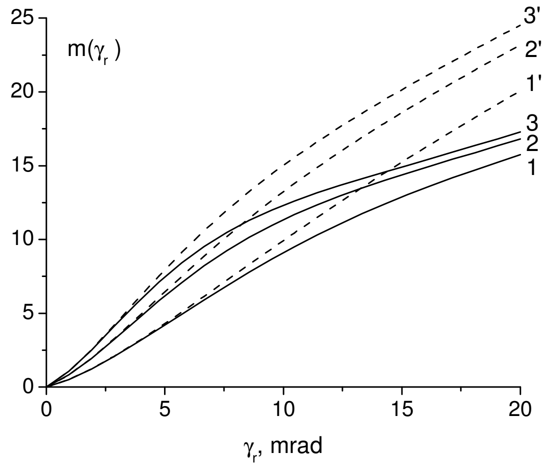

Fig. 2. The ratio $m(\gamma_r)$ calculated with (curves *1-3*) and without (curves *1'-3'*) the regard for the backscattering anisotropy, at the modal radius of the cloud particles $\bar{r}_s$ = 5.33 (*1, 1'*), 8 (*2, 2'*), and 10 (*3, 3'*) µm.

The dependences of ratio $m(\gamma_r)$ are presented in Fig. 2, corresponding to the specified model of cloudy medium. The input data for calculations are the following: the distance z = 2 km; the optical depth τ = 3, and the layer geometrical length is 1 km. From Fig. 2 it follows that failure to take into account the backscattering anisotropy results in the overestimated quantity of $m(\gamma_r)$. The influence of this factor grows with increase of the sizes of particles and optical thickness of a layer and becomes especially appreciable at FOV of the receiver γ_r > 5 mrad.

The following example relates to a problem of sea water sensing with the lidar located at a height of 300 m above the sea surface; the signal is detected from the depth of 20 m. We assume that the main contribution to scattering in sea water comes from suspended particles of two fractions: the finely dispersed fraction of mineral origin with sizes r < 1-2 μm (t – fraction) and the coarsely dispersed fraction of organic origin with sizes r > 1 μm (b - fraction) [4].

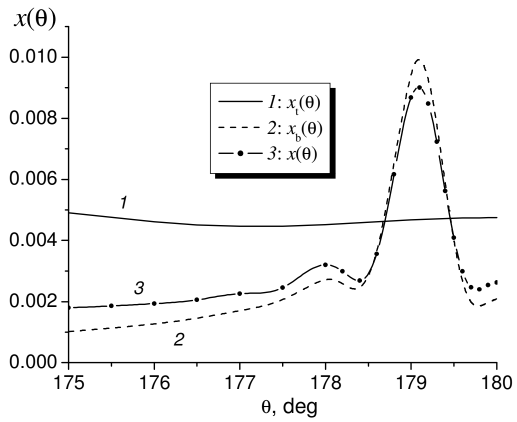

Fig. 3. Calculated scattering phase functions for the model of suspended particles in the sea water.

The typical dependences of scattering phase function $x_t(\theta)$, $x_b(\theta)$ and their weighted sum $x(\theta)$ are shown in Fig. 3 [7]. When modeling the optical characteristics of the suspension, the size distribution of t - fraction is described by the power law with the index ν= 4. The modified gamma-distribution with the modal radius $\bar{r}_s$ = 10 μm is chosen for modeling the size distribution of b-fraction. Curve *3* in Fig. 3 relates to the case when the contribution of t-fraction to the total scattering coefficient is equal to 20%.

As can be seen from Fig. 3, the scattering phase function $x_t(\theta)$ looks nearly isotropic one in the vicinity of a backward direction. On the contrary, the scattering phase function $x_b(\theta)$ has well-marked diffraction peak about the scattering angle θ = 179°, which is kept in the total scattering phase function. With increasing the sizes of particles of b - fraction their contribution to backscattering grows. At the same time, the amplitude of the diffraction peak is increased, and its position is shifted nearer to the scattering angle θ = 179°.

It is obvious, that the indicated factors cause the enhancement of the influence of the backscattering anisotropy on the angular behavior of the ratio $m(\gamma_r)$ with the growth of the particle modal radius $\bar{r}_s$ of b - fraction. It is confirmed by the results of calculations of ratio $m(\gamma_r)$, given in Fig. 4 for a layer with the optical

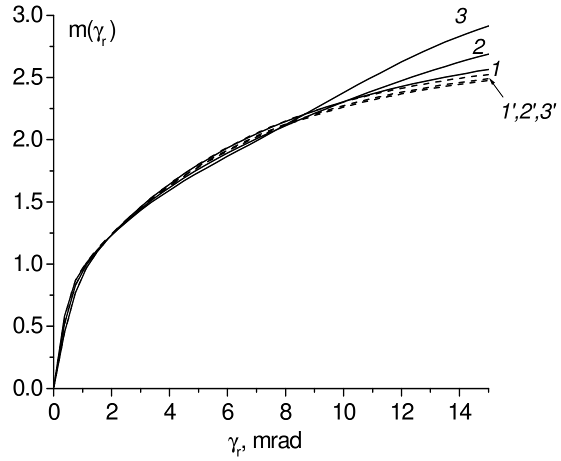

Fig. 4. The ratio $m(\gamma_r)$, calculated with (curves *1–3*) and without (curves *1'–3'*) the regard for the backscattering anisotropy, at the modal radius of the b-fraction particles $\bar{r}_s$ = 8 (*1, 1'*), 10 (*2, 2'*), and 12 μm (*3, 3'*).

depth τ = 2. In a given example, unlike cloud sensing, neglecting of the backscattering anisotropy results in underestimating the value of ratio $m(\gamma_r)$. This is observed for γ_r > 9 mrad and $\bar{r}_s$ > 10 μm.

REFERENSES

1. Dolin L.S., Savel'ev V.A. Characteristics of backscattering signal at pulse radiation of turbid medium by a narrow directional light beam, *Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana*,Vol. 7, No. 5, 505–510, 1971.
2. Bissonnette L.R. Multiple-scattering lidar equation, *Applied Optics*, Vol. 35, No. 33, 6449–6465, 1996.
3. Zege E.P., Katsev I.L., Polonsky I.N. Analytical solution to lidar return signals from clouds with regard to multiple scattering, *Appl. Phys.*, Vol. B60, 345–353, 1995.
4. Ocean optics, Vol. 1, Ed. by A.S. Monin, Nauka, Moscow, 1983.
5. Case K.M. and Zweifel P.F., *Linear transport theory*, Reading, Mass., Addison-Wesley Pub. Co., 1967.
6. Deirmendjian D., *Electromagnetic Scattering on Spherical Polydispersions*, American Elsevier Publishing, New York, 1969.
7. Veretennikov V.V. Effect of microphysical characteristics of suspensions on the backscattering characteristics of seawaters, *Atmospheric and Oceanic Optics*, Vol. 17, No. 9, 681-685, 2004.