CALCULATION OF LIDAR SIGNALS FOR HEXAGONAL ICE CRYSTALS

A.G. Borovoi, N.V. Kustova, and D.A. Dzhurmiy

Institute of Atmospheric Optics, Rus. Acad. Sci., Tomsk, Russia, e-mail:borovoi@iao.ru

ABSTRACT

The scattering or Mueller matrix for light scattering by ice crystal particles of cirrus clouds is conventionally calculated in the geometrical optics approximation. This leads to great uncertainties in the forward and backward scattering directions that are of main importance for lidar investigations. In this presentation, the Mueller matrix is calculated within the framework of physical optics where all uncertainties disappear because of accounting for wave phenomena. A new numerical method effectively accounting for diffraction has been developed and verified.

1. INTRODUCTION

Calculation of the radiative properties of cirrus clouds, which consist mainly of ice crystal particles, is a challenging problem for up-to-date atmospheric optics. These calculations are needed for interpretation of the data obtained by various remote-sensing techniques, including space-borne radiometers, ground-based and space-borne lidars, for incorporation in contemporary numerical climate models, etc.

Backscattering peak inherent to light scattering by ice crystal particles is the main obstacle for a reliable interpretation of lidar signals returned from crystal clouds. At present, no theoretical data describing this peak are known because of its singular character.

This peak was studied theoretically in our previous works within the framework of geometric optics [1-5]. The geometrical optics approach allowed us to clarify the physical regularities creating the backscattering peak for the conventional habit of hexagonal cylinders. In particular, our computer code generates all plane-parallel beams that leave a crystal of a given orientation. The shape, phase and polarization state of every beam are simply obtained. Then the main part of computer resources is spent for averaging these numerical solutions over possible crystal orientations. We proved that only a small part of orientations contributes effectively at backscattering, and this fact spared essentially the computer resources.

With the given parameters of the scattered beams, it would seem that the geometric optics solution could be easily generalized to physical optics to account for the wave phenomena, i.e. diffraction and interference. Indeed, a computer quickly calculates the Fraunhofer diffraction pattern from a plane-parallel beam since all the beams are of polygon shapes. Then a summation of the Fraunhofer patterns would lead to the desired physical-optics lidar returns. Unfortunately, as our experience showed, this direct method summarizing diffraction patterns reveals considerable difficulties. The reason is that the Fraunhofer patterns are quickly oscillating functions with various spatial frequencies and with large dynamic ranges. Therefore it is impossible to choose common numerical grids satisfying all constituents.

To overcome this obstacle, a new method for numerical calculation of diffraction has been developed. This method was firstly considered in [6] for the case of a particle with a fixed orientation. The idea of the method is to replace the diffraction patterns that are difficult objects for numerical calculation by their Fourier transforms called the shadow function. Unlike the diffraction patterns, the shadow functions have the following advantages. First, they have a simple geometrical meaning, and this fact allows us to control the numerical data visually. Second, they are strictly equal to zero outside a finite domain of the size of a scattering particle. Third, these values are smooth functions with the dynamic range within the interval [0, 1]. Fourth, they don't depend on incident wavelength.

In this contribution, the method of the shadow functions is applied to calculate the backscattering Mueller matrix for randomly oriented hexagonal ice columns and plates. Then this matrix is used for an estimation of real lidar returns.

2. METHOD OF SHADOW FUNCTIONS IN THE THEORY OF FRAUNHOFER DIFFRACTION

If a plane-parallel electromagnetic wave is incident on an ice crystal of cirrus clouds, the scattered light leaving the crystal surface consists of a lot of plane-parallel beams. Every beam is characterized by its transverse polygon shape, phase and state of polarization. At large distances from the particle, these beams are spread over scattering angles near the initial propagation direction because of diffraction. If the diffracted beams are overlapped in space, they also interfere with each other. Taking into account both diffraction and interference, we get the desired scattered field within the framework of physical optics.

Shape of a plane-parallel beam leaving a crystal surface is determined by the indicator function

$$\eta(\boldsymbol{\rho}) = \begin{cases} 1 & \textit{inside the beam} \\ 0 & \textit{outside} \end{cases} \qquad (1)$$

where $\boldsymbol{\rho}$ are coordinates in the plane that is perpendicular to the propagation direction. At far distance from the crystal, the plane-parallel beam is transformed into a divergent spherical wave that is spread near the initial propagation direction according to the Fraunhofer diffraction equation

$$f(\mathbf{n}) = \frac{k}{2\pi i}\int \eta(\boldsymbol{\rho})\exp(-ik\mathbf{n}\boldsymbol{\rho})\mathrm{d}\boldsymbol{\rho} \qquad (2)$$

where $k=2\pi/\lambda$, λ is the wavelength, $\mathbf{n}$ is the projection on the ρ-plane of scattering direction Ω ($|\Omega| = 1$).

In optics, not the complex-valued quantity of Eq. (2) is measured experimentally but its quadratic value called the diffraction pattern

$$I(\mathbf{n}) = |f(\mathbf{n})|^2 \qquad (3)$$

Since the diffraction pattern has a lot of drawbacks hampering its numerical calculations, we introduce the 2D Fourier transform of the diffraction pattern

$$S(\boldsymbol{\rho}) = \int I(\mathbf{n})\exp(ik\mathbf{n}\boldsymbol{\rho})\mathrm{d}\mathbf{n} \qquad (4)$$

that proves to be equal to

$$S(\boldsymbol{\rho}) = \int \eta(\boldsymbol{\rho}')\eta(\boldsymbol{\rho}' - \boldsymbol{\rho})\mathrm{d}\boldsymbol{\rho}' \qquad (5)$$

This function is called the shadow function. Its general properties are discussed elsewhere [6].

In the procedure of averaging over particle orientation, we have to summarize the Stokes parameters of the scattered fields or, equivalently, the Mueller matrix. Since size of crystal particles are much larger than wavelength, the diffraction phenomena can be included by the scalar factors (2)-(4). So, we can consider scalar values, and polarization of light can be accounted for afterwards.

3. FORWARD SCATTERING

Since crystal particle of cirrus clouds are much larger than visible wavelengths, small-angle scattering can essentially impact on lidar signals returned from cirrus clouds because of small-angle multiple scattering along the light path. In this Section, the method of shadow functions is applied to small-angle scattering.

For small-angle scattering, two kinds of beams appear. The first one corresponds to diffraction on contour of a crystal particle, and its indicator function (1) is nonzero inside a particle projection on the plane that is perpendicular to incident direction. The shadow function (5) depends on both absolute size of a particle and on particle shape. It is convenient to put away the absolute particle sizes and to consider only particle shapes by means of the following normalization

$$S_0(\mathbf{R}) = < S(\boldsymbol{\rho}/\sqrt{< s >}) > / < s > \qquad (6)$$

where s is area of particle projection, <…> means the statistical averaging, and $\mathbf{R}$ is the dimensionless 2D variable. So, this normalized shadow function depends on only particle shape, and it has the following properties

$$S_0(0) = 1 \quad \int S_0(\mathbf{R})d\mathbf{R} = < s^2 > / < s >^2 \qquad (7)$$

In atmospheric optics, two cases are of interest. Fist case corresponds to horizontally oriented particles with random orientation relative to the azimuth angles. Fig. 1 shows the shadow functions calculated for the case of vertical incidence of light on horizontally oriented hexagonal ice columns with various aspect ratio Q = diameter/length. The columns were assumed to be Parry oriented, i.e. the columns don't rotate relatively their main axes. In this case, the second value of Eq. (7) is equal also to unity. The limiting curve in Fig. 1 corresponds to a sphere. The more shape deviate from a sphere, the more its shadow function deviate from the limiting sphere curve.

Fig.2 presents the same curves for 3D randomly oriented hexagonal ice columns. We see the same deviations from the limiting curve of a sphere. Let us emphasize that the shadow functions are convenient and physically obvious values for approximation of complicated particle shapes by simpler shapes, for example, by spheres.

In addition to the diffraction caused by particle contours, small-angle scattering includes also the beams formed by parallel facets of crystals. Indeed,

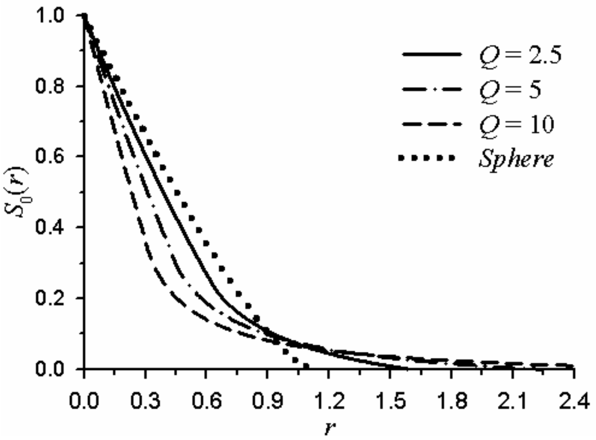

Fig.1. The shadow functions for horizontally Parry oriented hexagonal ice columns with various aspect ratios Q, where $r = |\mathbf{R}|$.

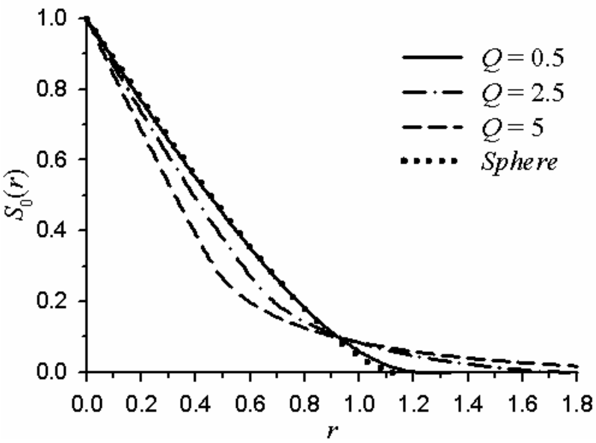

Fig. 2. The same as in Fig.1 for 3D randomly oriented hexagonal ice columns.

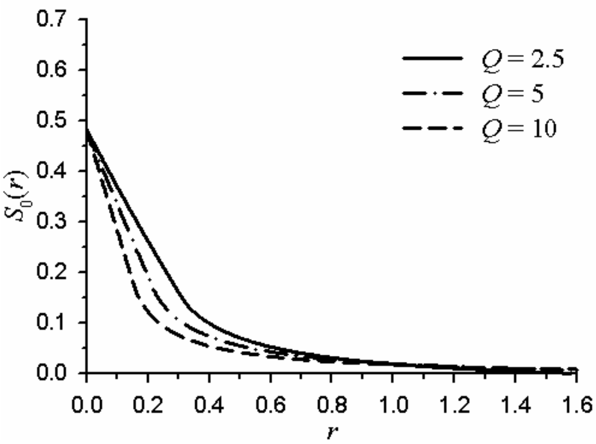

Fig.3. The shadow functions of the transmitted beams for horizontally Parry oriented hexagonal columns.

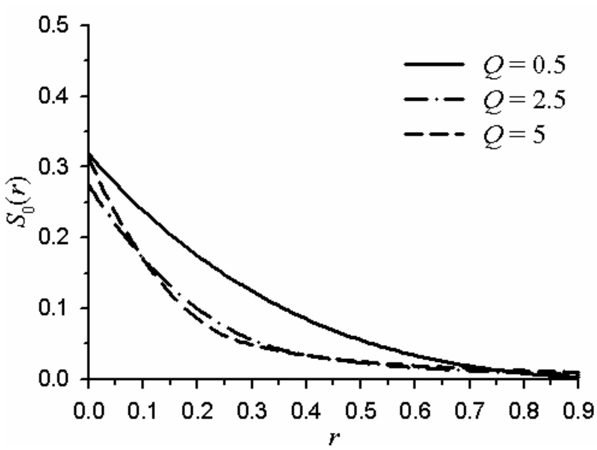

Fig.4. The same as in Fig.3 for 3D randomly oriented hexagonal columns.

parallel facets of a crystal are equivalent to a plane-parallel plate, and they produce the transmitted beams leaving the crystal in the forward direction. In this contribution, we have calculated exactly the transmitted beams for the first time. Fig..3 and Fig.4 presents the shadow functions for the transmitted beams for Parry and 3D randomly oriented crystal, respectively. Thus, small-angle scattering by hexagonal ice columns is completely determined by the shadow functions presented in Figs. 1-4.

4. BACKSCATTERING

Lidar signals are mainly determined by backscattering. As was mentioned above, calculation of the backscattering Mueller matrix for randomly oriented ice crystal particle is a challenging problem of up-to-date atmospheric optics that is not satisfactory solved yet. The direct calculation of this matrix is extremely costly for computers of moderate characteristics. This problem is overcoming in this presentation due to the method of shadow functions.

Figs. 5 and 6 shows the shadow functions calculated for a hexagonal column and plate, respectively, with a fixed particle orientation. However, a statistical averaging of such patterns demands a considerable amount of computer time. At present, such calculations are processing. The numerical results obtained will be presented at the 23rd ILRC conference. A comparison of these results with the available experimental data will be presented as well.

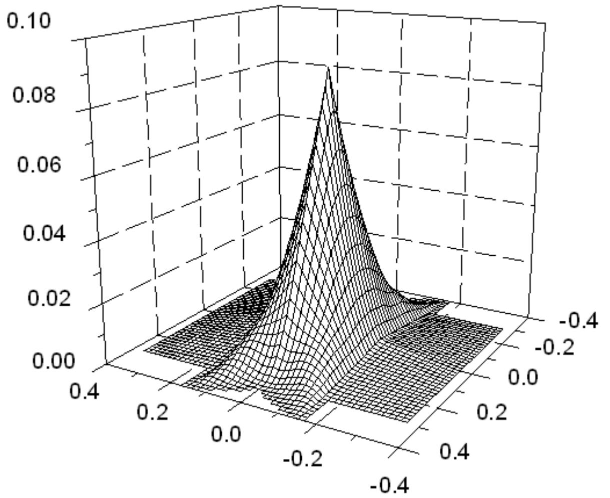

Fig. 5. The backscattering shadow function for a hexagonal ice column of the aspect ratio of 2.5 with a fixed orientation (the angle between the main axis and incident direction is equal to 32°). The shadow function is normalized according to Eq. (6). Here, the vertical axis shows its value and the left and right horizontal axes correspond to the dimensionless 1D variables in the longitudinal and transverse directions, respectively, relative to the plane formed by the incident direction and the main crystal axis.

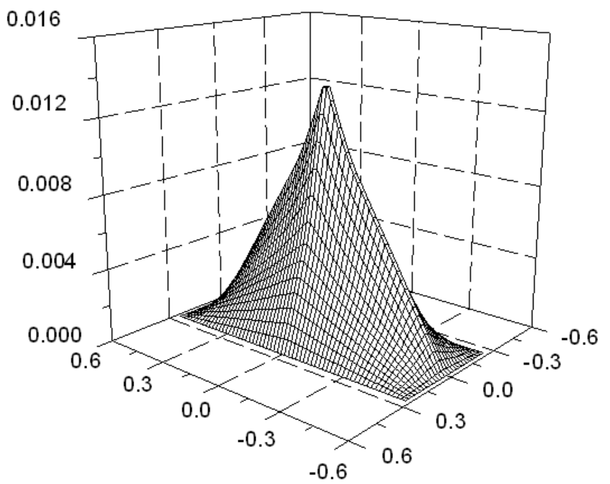

Fig.6. The same as in Fig. 5 for a hexagonal plate of the aspect ratio of 0.5 tilted at 58°.

5. CONCLUSIONS

Though the scattering or Mueller matrix for ice crystal particles of cirrus clouds has been calculated more or less exactly at the lateral scattering direction, there are great uncertainties at the forward and backward directions because of singular magnitudes of the matrix in these directions. They are the directions that are of main importance for lidar investigations. The method of the shadow functions developing by the authors is a promising tool to solve the problem numerically in the nearest future.

6. ACKNOWLEDGMENTS

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