IDENTIFICATION OF STATIONARY STATES USING LOGARITHMIC AVERAGES

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ABSTRACT

Signal averaging is a standard means of improving the signal to noise ratio during lidar signal acquisition. The underpinning assumption in such an operation is that the system under observation is stationary and ergodic, i.e. the statistics of the system under observation 1) do not change within the observation period and 2) the temporal average is a good indicator of the ensemble average.

Interpretation of data averaged under highly dynamic atmospheric conditions may thus prove to be problematic unless individual waveforms are accessible for reference and interpretation –the preferred situation. However, some level of local data processing and reduction may be considered for remotely operating or autonomous systems, such as for bandwidth restricted space missions, even though averaged data in such cases can be very misleading.

We discuss an approach in which the geometric mean and error, together with arithmetic mean and error, can be used to establish the validity of averages without recourse to the individual waveform data from which these values are reduced.

1. INTRODUCTION

In some lidar operational circumstances, particularly space missions, data flow can be constrained by the bandwidth of the communication link. One potential solution to this problem is on-board data accumulation – i.e. the addition of multiple waveforms per integration interval– and processing. The potential advantage offered by signal on-board signal averaging –i.e. reduction of data volume with an improved signal to noise ratio– is lost as the system under investigation departs from stationary ergodicity, as shown in Fig. 1.

The curves represent continuous, infinitely-resolved signal distributions at a given range in their large sample limit. The signal strength, notionally in volts, is varies along the abscissa. The sample frequency is plotted against the ordinate. The top curve represents a signal distribution in which the scattering system is stationary and ergodic i.e. 1) the statistics of the system under observation do not change within the observation period and 2) the temporal average is a good indicator of the ensemble average. The arithmetic average, represented by the vertical line, is coincident with the signal distribution mode. In the middle panel, two scattering populations, similar but not quite identical, are present. The modes of the individual distribution are depicted by solid vertical lines and the respective distributions by the solid curves. The average is represented by the dotted vertical line and the dotted curve represents the signal distribution as inferred from the standard deviation calculated from the bimodal distribution data. The average is roughly representative of the system state, but with some loss of information. The bottom panel depicts a situation in which two distinct scattering populations are present during the averaging period, such as cloud returns under partially cloudy conditions. The signal distribution inferred from a simple average completely misrepresents the reality of the scattering conditions.

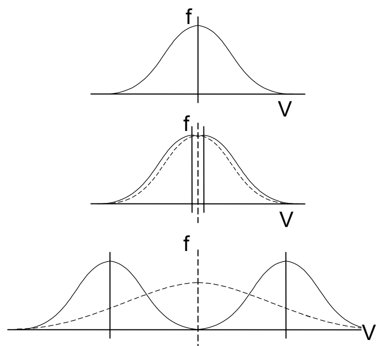

Fig. 1: Averages and Inferred Distributions for Bimodal Distributions

In such widely varying circumstance, a large error interval is to be expected, also as seen in the lower panel of Fig. 1. However, the size of the error interval alone is not a reliable indicator of non-stationary conditions as it does not distinguish between multimodal distributions and broad, unimodal scattering distributions. Similarly, closely situated scattering distributions would go undetected within the distribution inferred from the average and standard deviation, as seen in the middle panel of Fig. 1, without even the size of the error interval to indicate that the conditions may be non-stationary/ergodic.

Ambiguous interpretation of such processed data originates in the equal weight ascribed to all contributing data. Logarithmically collected data, as implemented with logarithmic amplifiers (logamps) are inherently skewed towards the lower values in the distribution [1]. This feature may be usefully employed to distinguish between unimodal and multimodal signal distributions, as might occur when signals are averaged over a time period in which scatterers, such as clouds, are in and out of the receiver field of view during the integration period

Accumulated logarithmic signals are reduced as the so-called geometric mean. The geometric mean is insensitive to the presence of large-valued outliers and is always less than or equal to the arithmetic mean [2]. The more tightly clustered the data distribution, the smaller the difference between the geometric and arithmetic means. The geometric mean is the inversion of the raw digitized logamp signal by means of the antilog operation, described in Eq. 1:

$$\begin{aligned} \operatorname{anti}\log\left(\bar{x}_{raw}\right) &= 10^{\frac{\log\left(\prod_{j=1}^{M}\prod_{i=1}^{N} x_{ij}\right)}{M\cdot N}} \\ &= \sqrt[(M\cdot N)]{\prod_{j=1}^{M}\prod_{i=1}^{N} x_{ij}} = \bar{x}_g \end{aligned} \tag{1}$$

where $\bar{x}_{raw}$ is the arithmetic average of the raw logamp signal, x_{ij} is the ith logarithmic signal collected at the jth range interval, and N and M are the number of temporal and spatial samples, respectively.

The geometric standard deviation of the geometric mean is defined in Eq. 2: The error limits of the logarithmic signal inversion are defined in Eq. 3:

Unlike the arithmetic standard deviation, the geometric standard deviation is a factor applied to the geometric mean in order to determine the upper and lower error limits.

$$\begin{aligned} \sigma_g &\equiv \operatorname{anti}\log\sqrt{\frac{\sum_{j=1}^{M}\sum_{i=1}^{N}\left(\log\left(x_{ij}\right)\right)^2 - \frac{\left(\log\left(\prod_{j=1}^{M}\prod_{i=1}^{N} x_{ij}\right)\right)^2}{N\cdot M}}{(N-1)\cdot(M-1)}} \\ &= 10^{\sigma_{raw}} \end{aligned} \tag{2}$$

$$\begin{aligned} x_{g_u} &= 10^{\left(\bar{x}_a+\sigma_a\right)} = 10^{\left(\log\left(\bar{x}_g\right)+\log\left(\sigma_g\right)\right)} = \left(\bar{x}_g\cdot\sigma_g\right); \\ x_{g_l} &= 10^{\left(\bar{x}_a-\sigma_a\right)} = 10^{\left(\log\left(\bar{x}_g\right)-\log\left(\sigma_g\right)\right)} = \left(\frac{\bar{x}_g}{\sigma_g}\right) \end{aligned} \tag{3}$$

The bias between signal data retrieved linearly and with logarithmic compression is expressed in Eq. 4 simply as:

$$Bias = \left(\frac{\bar{x}_a - \bar{x}_g}{\bar{x}_a}\right) \tag{4}$$

where $\bar{x}_a$ is the arithmetic average of linearly accumulated data. The geometric mean is always less than or equal to the arithmetic mean, i.e. the minimum value of (4) is zero. The maximum value is one.

When the signal distribution becomes multimodal, two things are thus expected to happen 1) a bias arises between data reduced through the geometric mean and those reduced through the arithmetic mean. This is a corollary of the arithmetic- geometric mean inequality; 2) the inversion of the geometric mean results in an increasingly skewed signal distribution.

These attributes are investigated for their usefulness in establishing stationary ergodic conditions and the validity of averaged data.

2. DISCUSSION

Fig. 2 depicts signals accumulated from a small breadboard lidar system built by Optech Incorporated. The lidar is zenith sounding. The data shown were collected using 1064 nm output with 300 μJ per pulse at 100Hz. The signal returns displayed in Fig. 2 are averages over 348 shots at an altitude of about 1.58–1.7 km, collected in Toronto on April 5, 2005 under mostly cloudy conditions. The return signals were sampled at 2.5 m intervals.

The heavy dashed black curve depicts the arithmetic and geometric mean signal strengths as a function of range. They are not identical, but on the scale of the chart are indistinguishable. The bias between them is depicted by the heavy black line, with a negative deflection. These two curves are described by the left-hand y-axis The arithmetic relative error is depicted by the solid black line. The arithmetic relative error with DC offset eliminating the residual error is depicted by the thin dashed black line.

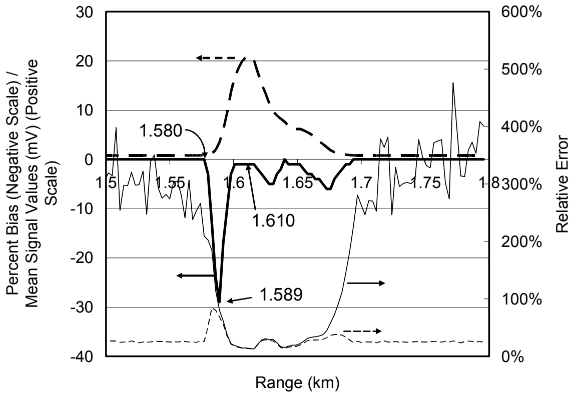

Fig. 2: Arithmetic/Geometric Mean, Bias and Relative Errors per Range; Heavy Dashed Black Line – Arithmetic/Geometric Mean, Heavy Solid Black Line – Arithmetic/Geometric Mean Bias, Thin Solid Black Line – Relative Error without DC Zero Offset, Thin Dashed Black Line – Relative Error with DC Zero Offset

Both the arithmetic and geometric mean value signal strengths rise from initial values at 1.580 km altitude increase to a peak value at an altitude of 1.610 km and then tapering off until 1.691 km. There is a very close overlap between the geometric and arithmetic mean signal whenever the signal strength reaches a peak or relative plateau. The arithmetic-geometric mean bias is greatest in those ranges where the signal strength is changing most rapidly and least in those ranges where the signal strength reaches an extremal value or plateau. Three features of particular interest in this respect and are range-labeled 1) the onset of the signal return at 1.580 km, 2) the maximum magnitude of bias between the geometric and arithmetic means at 1.589 km and 3) the region of least bias between the arithmetic and geometric means at 1.610 km. The raw data from which these features are developed are instructive.

Fig. 3 depicts a histogram of signal strength values returned clouds from an altitude of 1.580 km. The solid black histogram depicts data obtained with a logarithmic amplifier and subsequently digitized. The dashed curve represents the inversion of the logamp histogram, i.e. each logarithmic signal return is inverted individually and digitized according to the linearly-defined voltage levels, replicating linear capture of the signal.

There is a qualitative difference between the two histograms. The raw logamp signal return is multimodal and is distributed predominantly about a peak at 200 mV, but with many excursions towards larger signal strength values, some of which are more than 500 mV. Noise in the lower bits largely accounts for skewing of the distribution to the right [3]. Small signals embedded in this regime can be retrieved through averaging.

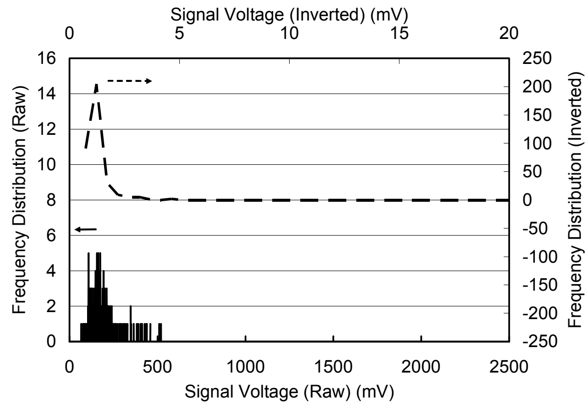

Fig. 3: Raw Logamp Signal Histogram and Inverted Signal Distribution, Clouds at 1.58 km Altitude

The inverted logamp signal, however, is a simple unimodal distribution. Large signal contributions (~ 500 mV), seen in the raw signal histogram, are lost in the inversion process. Linearly accumulated data have coarser resolution in the small signal regime than data accumulated logarithmically. Linearly accumulated data thus closely resemble the inverted logamp results.

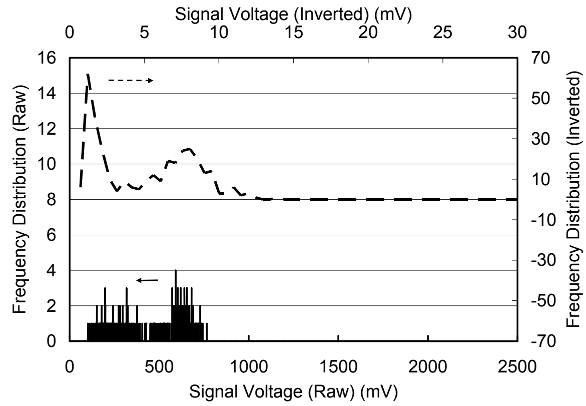

Fig. 4: Raw Logamp Signal Histogram and Inverted Signal Distribution, Clouds at 1.589 km Altitude

Fig. 4 depicts a histogram of the raw logamp data retrieved from an altitude of 1.589 km, the range where the bias deflection is greatest. The data distribution in Fig. 4 is multimodal, with at least two local peaks in the raw logamp data. The multimodal structure is also evident in the inverted logamp signal, but the large positive logamp values are de-emphasized in the inversion. The large-valued inverted signal distribution is spread out while the low signal distribution appears to be largely lumped into the smallest voltage levels. These results are consistent with the situation described in the lower panel of Fig. 1. The situation is non-stationary ergodic and the temporal average contains no further information about the ensemble.

Fig. 5 shows the data distribution corresponding to the peak values of the average profiles, where the bias is small. The distribution of the raw logamp signal is tightly distributed upon a central limit. The inverted logamp signal distribution reverts to a symmetric, centrally-limited envelope with good fidelity. Of the three distributions presented so far, the data distribution shown in Fig. 5 represents the most appropriate candidate data set for averaging.

Review of similar data from the data set shows that the magnitude of the bias between the arithmetic and geometric means is associated with the multimodal signal distributions, which indicate non-stationary/non-ergodic conditions in the scattering medium. Thus low bias regions appear to be the most appropriate for signal averaging.

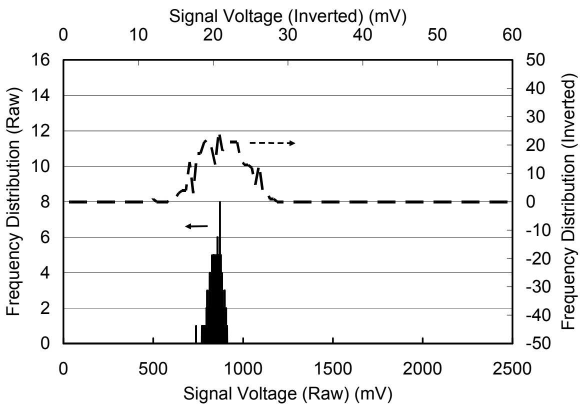

Fig. 5: Raw Logamp Signal Histogram and Inverted Signal Distribution, Clouds at 1.61 km Altitude

Logarithmically reduced data may be used to correctly identify valid averaging conditions without the presence of a linear amplifier circuit. Local extremal points from the plot of the geometric mean presented in Fig. 2 were assessed for their bias as well as the relative geometric error, defined as the difference between the upper and lower geometric error limits divided by the geometric mean. The results are plotted in Fig. 6.

The black diamonds are from low bias data at ranges of 1.610, 1.640 and 1.691 km. The black dots are from high bias data at ranges of 1.580, 1.589, 1.628 and 1.673 km. The horizontal dotted line indicates the limit of bias attributable to discretization of the logarithmic signal (<3%) [4]. The correlation between the bias and the relative error is depicted by the solid and dashed lines, accounting for the entire set of data and that for which the two outlying points are omitted from the calculation. There is strong correlation between the bias and the geometric relative error: the geometric relative error is a potential indicator of stationary-ergodic conditions.

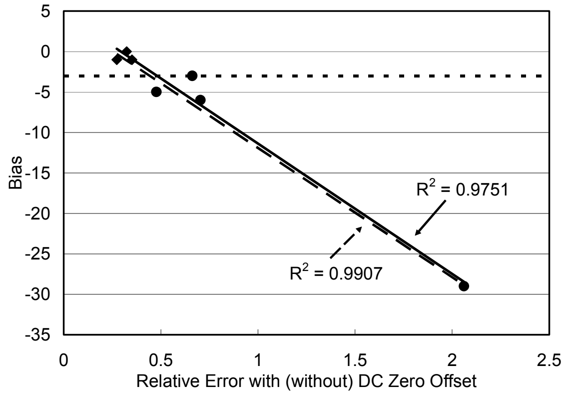

Fig. 6: Arithmetic-Geometric Mean Bias versus Geometric Relative Error. Diamonds-Low Bias Data, Circles-High Bias Data, Solid Black Line-Trend (All Data), Dashed Black Line- Trend (No Outliers), Gray Line- 3% Bias Cut-Off

3. CONCLUSIONS

Small bias values between the arithmetic mean – corresponding to data collected with a linear amplifier– and geometric mean –corresponding to data collected with a logarithmic indicate amplifier– indicate approximately stable conditions and signal averages are valid.

Large biases between the arithmetic and geometric mean occur when the scattering medium is unstable and where averaged signal data are not interpretable

The bias is correlated to the geometric mean relative error, i.e. the ratio of the geometric mean error interval to the mean signal value. The relative error as determined through the use of the geometric standard deviation and mean is a good predictor of bias and tends to eliminate or reduce the effect of data outliers. The relative error as determined by the arithmetic standard deviation and mean is not a good predictor of bias and data outliers can greatly affect results.

4. REFERENCES

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